Investing in Misallocation *

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Abstract

We document that 20% of Compustat firms have above-median investment rates despite having below-median marginal product of capital (MPK), seemingly "misallocating" productive resources. These firms are typically younger and significantly more likely to experience a substantial upward jump in their sales and MPK in the following years. They account for a significant share of innovative activity and their investments predict future aggregate productivity in the economy, creating value in ways not captured by their MPK. We propose and estimate a simple endogenous firm growth model that captures the key features of the cross-section of firms and allows for counterfactual analysis. Hypothetical firm investment policies that ignore the potential for future jumps reduce MPK and investment dispersion but also lower aggregate productivity.

JEL classification: D21, D22, D24, D25, E22, G31, O30, O47.

Keywords: Misallocation, MPK, endogenous growth, investment, jumps.

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1 Introduction

Recent empirical studies have uncovered significant heterogeneity in productivity levels across firms, even within industries. This finding is often interpreted as evidence of an inefficient capital allocation among firms, stemming from various distortions, and is referred to as "misallocation" (Hsieh and Klenow, 2009). The literature has focused on understanding the role of specific factors, such as adjustment costs, financing frictions, and firm risk in hindering the equalization of productivity across firms. A key insight emerging from this body of research is that reallocating capital from firms with lower marginal productivity to those with higher marginal productivity, such as by lifting financial constraints, can result in improvement in overall output and efficiency while reducing cross-sectional dispersion in productivity.

From the perspective of misallocation, which is usually measured by the dispersion in firms' value added to input ratios, firms that make large investments despite having low output can be seen as recipients of misallocated capital. However, many well-known firms, such as Tesla or Amgen, adopted a strategy of making substantial investments for several years after going public, even though their output levels were initially low. This approach ultimately led to a significant increase in their sales and productivity. This observation challenges the notion of the initial "misallocation" of capital, as it suggests that such investments were, in fact, successful strategies. Reallocating capital away from these firms, based solely on their lower productivity at the time, could potentially undermine long-run efficiency and hinder overall productivity growth.

In this paper, our focus is on firms that appear to be accumulating too much capital relative to their output, i.e. "investing in misallocation", relative to other firms in the cross-section. We study the properties of these firms and explore their role for the dispersion across firms and aggregate efficiency. Empirically, we document that 20% of firms in the Compustat data set simultaneously have below-median marginal product of capital (MPK) and above-median investment rates. These firms are typically younger and approximately 4% of them experience a large upward jump in their sales and MPK in the subsequent period, while the remaining firms have a much lower probability of experiencing such jumps. Importantly, these firms contribute significantly to the innovative activity within the economy. They exhibit a patent issuance rate that is more than twice as high as other firms, and their rates of breakthrough patent issuance and patent citation are more than three times as large. They are also more likely to be in the innovation stage along their product life cycle.

We explore whether the occurrence of infrequent but significant growth spurts, characterized as jumps, can rationalize the presence and characteristics of high investment-low MPK firms. To accomplish this, we propose and estimate a simple model that incorporates heterogeneity among firms, capturing key empirical observations. In our model, firms are classified as either high-type or low-type, representing their differing potential for productivity growth. Unlike low-type firms, high-type firms have the potential for a substantial positive jump in productivity, and the likelihood of experiencing such a jump increases with their level of investment.[1] In our framework, high-type firms optimally invest heavily to maximize their chances of a jump occurring and to be prepared for its arrival, rather than solely aiming for immediate production benefits. The presence of high-type firms disrupts the tight connection between MPK and investment in the cross-section, leading to a realistic proportion of firms exhibiting high investment despite having low MPK levels.

We conduct a counterfactual experiment where firms' investment policies ignore the potential of future jumps even though jumps are still present in the data-generating process. That is, high-type firms do not engage in "investing in misallocation" and just invest the same amount as a low-type firm with the same productivity and capital. In the counterfac-

[1] In our model, we consider a single type of capital, which encompasses physical capital such as equipment as well as intangible capital such as innovative capacity, organizational capital, brand capital, customer base, etc.

tual model, investment and MPK become closely aligned in the cross-section, resulting in the absence of firms in the high investment-low MPK portfolio. This contradicts the empirical evidence where MPK dispersion among high investment firms is as high as it is among all firms. Furthermore, the overall MPK and investment dispersion becomes lower suggesting a decline in misallocation. However, this removal of "investing in misallocation" also leads to lower productivity in aggregate demonstrating that eliminating this source of cross-sectional MPK dispersion does not result in higher efficiency. Consistent with the lower aggregate productivity in the model counterfactual, our empirical findings reveal that the median investment rate among firms in the high investment-low MPK portfolio serves as a robust predictor of future aggregate growth in total factor productivity (TFP). Specifically, a one standard deviation increase in the investment rate corresponds to a one standard deviation increase in 5-year TFP growth. In sum, while traditional metrics may classify these firms as misallocating resources, they are creating value in ways that are not captured by their current MPK levels.

The influential works of Hsieh and Klenow (2009) and Restuccia and Rogerson (2008) introduced heterogeneous distortions to input and output prices faced by firms, but did not explicitly identify the sources of misallocation. Subsequent research has identified various types of distortions as possible sources of misallocation, including adjustment costs and volatility (Asker, Collard-Wexler, and De Loecker, 2014), information frictions (David, Hopenhayn, and Venkateswaran, 2016), and excess investor demand (Choi, Kargar, Tian, and Wu, 2023). The common thread in this literature is the exploration of the factors underlying the deviations of firm-level capital from an efficient allocation across firms, leading to disparities in MPK and productivity losses.

Another prominent strand of literature has proposed financial frictions as a significant factor contributing to misallocation (Midrigan and Xu, 2014; Moll, 2014; Whited and Zhao, 2021; Bau and Matray, 2023). These studies emphasize the importance of financial constraints that impede firms, especially those with high MPK and low investment, from accessing an optimal level of capital. Notably, financial frictions tend to have a particularly pronounced impact on innovative firms. Li (2011) asserts that financial constraints are more likely curtail the investment opportunities of these firms, which face additional financial constraints due to information asymmetry and agency problems (Hall and Lerner, 2010) as well as low asset tangibility (Almeida and Campello, 2007). Drawing on the insights derived from our model and the empirical evidence, we posit that within an environment characterized by financial frictions, efficient resource allocation following the elimination of these constraints may require a higher allocation of resources to low MPK but innovative firms that exhibit the potential for rapid future growth, thereby further increasing the dispersion in MPKs.

To distinguish and quantify the role of various channels on misallocation, David and Venkateswaran (2019) employ a framework that accounts for adjustment costs, information frictions, and firm-specific factors to capture all remaining drivers of investment decisions. Their findings indicate a significant influence of highly persistent firm-specific factors on investment decisions. Subsequently, David, Schmid, and Zeke (2022) demonstrate that differences in firm risk premia can create a persistent firm-specific factor that drives a wedge in firms' investment decisions, potentially explaining part of the observed misallocation. Our model generates a similarly persistent wedge in investment patterns due to firms' heterogenous growth prospects, resulting in persistently different MPKs. However, our mechanism operates differently from the risk premium channel in that the presence of risk premium heterogeneity increases MPK dispersion and lowers aggregate productivity, while "investing in misallocation" increases both dispersion and productivity.

Our study challenges the conventional notion that high productivity dispersion always indicates inefficient outcomes. We find that investments made in anticipation of potential rapid growth, whether in equipment, software, R&D, or other types of intangibles, play a crucial role in fueling economic growth, despite the initial appearance of misallocated capital. Therefore, our results align with the findings of Haltiwanger (2016) and Haltiwanger, Kulick, and Syverson (2018), who emphasize that conventional measures of misallocation may not only identify spurious inefficiencies but are also more likely to do so among firms that possess unique strengths, such as future demand and customer base. Our work provides compelling evidence that high investment-low MPK firms exhibit superior future productivity prospects, whether driven by technological advancements and innovation or by strong demand dynamics.

While the elimination of most mechanisms for misallocation generally leads to increased efficiency by reducing dispersion (such as easing financial constraints, minimizing risk premium dispersion, or lowering adjustment costs), Kehrig and Vincent (2020) present an exception to this pattern. They demonstrate that within-firm dispersion, specifically MPK dispersion across establishments, can enhance efficiency. Their model focuses on firms with multiple establishments subject to fixed adjustment costs and financial frictions, highlighting the benefits of this "good" dispersion. In our research, we explore a different source of "good" dispersion, namely the investment choices made by firms based on their growth prospects. Our findings indicate that allowing firms to make such investment decisions can contribute to a higher level of efficiency, even in the presence of dispersion.

Our paper is informed by the extensive literature on endogenous firm growth, which investigates how innovation can enhance productivity and competitiveness, leading to growth, increased profits, market share, and overall technological advancement. We connect to this vast and continually expanding literature in various ways. In particular, our paper heavily relies on the works of Klette and Kortum (2004) and Acemoğlu, Akçiğit, Alp, Bloom, and Kerr (2018), as we introduce exogenous heterogeneity in firms' productivity growth intensity, emphasize how this intensity increases with firm investment and capital accumulation, and model how it changes throughout the firm's life cycle. Although we recognize the importance of R&D and innovation – the exclusive focus of this literature – that can lead to rapid growth in the future, our empirical findings suggest that both intangible investment such as R&D and innovation and physical investment such as equipment and software are associated with higher likelihood of jumps. Hence, we take a reduced-form approach in modeling jumps with a single type of capital and remain agnostic about the further micro level drivers of jumps within firm investment.

Furthermore, our work shares a connection with Acemoğlu, Akçiğit, Alp, Bloom, and Kerr (2018) in that they expand on the concept of misallocation beyond conventional productive inputs to include R&D inputs. Their research illustrates that reallocating innovative resources, such as skilled R&D workers, from less innovative established firms to younger, innovative firms can lead to significant welfare gains, which is in line with our results. This finding underscores the importance of adopting a more comprehensive perspective on misallocation, rather than solely focusing on realized MPK.

Our paper is structured as follows. Section 2 presents the motivating empirical evidence. Section 3 presents our simple model of firms. Section 4 discusses the model's fit to the data and explores various quantitative aspects of the model and data. Section 5 offers insights into the economic mechanism by analyzing portfolio equity returns. Section 6 provides concluding remarks.

2 Motivating evidence

This section introduces the empirical evidence that provides the basis for our subsequent model and quantitative evaluation.[2] We begin our discussion by establishing a connection

[2]Detailed information about the data and variable construction is provided in Appendix A.1.

between the probability of firm jumps (large increases in output and marginal product of capital), investments, and the current marginal product of capital. “Jumps” are defined as instances where a company’s sales more than double, while its marginal product of capital increases by at least 50%.[3] From 1975 to 2015, the unconditional annual probability of a firm experiencing a jump was 1.5%. Although jumps were infrequent on an annual basis, many Compustat firms encountered a performance jump at some point during their existence. Among firms that joined Compustat after 1975 and remained for at least 5 years, 13% experienced at least one jump until 2015.

Next, we explore the relationship between firm characteristics and the probability of jumps according to our defined criteria. The purpose of this empirical analysis is to examine whether the distribution of capital across firms is linked to the occurrence of large booms at the individual firm level. Specifically, we investigate whether the likelihood of jumps is influenced by the marginal product of capital, and whether firms allocate more capital to current investments in anticipation of future jumps.

Table 1 presents the parameter estimates obtained from a linear probability model that investigates the relationship between experiencing a jump and several variables, including lagged investment rates (I/K) in physical and intangible capital, lagged marginal product of capital (MPK), and firm age. In Column 1, we find that firms with initially low MPK can experience a significant increase in both MPK and sales. This suggests that MPK is not a fixed characteristic of a firm and can vary over time.[4] Furthermore, Column 2 shows that investment in both physical and intangible capital predicts a higher likelihood of a jump occurring. In Column 3, which includes both MPK and investment measures, all variables demonstrate strong predictive power. Specifically, given a certain level of current

[3]To minimize the impact of noise, we measure jumps over a four-year period where the thresholds are applied to the growth of sales and MPK from the first two years to the last two years. All results are robust to variations in cutoffs for sales and MPK growth. Further details can be found in Appendix A.1.

[4]We include year×industry fixed effects in these regressions. Therefore, all comparisons are across firms within the same industry–year pair.

Table 1: Determinants of Firm Jumps

This table presents coefficient estimates obtained from linear probability models analyzing realized jumps. The dependent variable is the jump dummy, which takes a value of 1 when a jump occurs from the current period to the next period. Jumps are defined as cases where a company's sales double, accompanied by a minimum 50% increase in its MPK. For a comprehensive explanation and the definition of explanatory variables, please refer to Appendix A. The regressions incorporate 2-digit SIC industry-year fixed effects. The corresponding *t*-statistics are presented in parentheses, and standard errors are clustered at the firm-year level. Statistical significance levels are indicated by one, two, and three stars, denoting significance at the 10%, 5%, and 1% levels, respectively. The variable N denotes the count of firm-year observations, while R^2 represents the adjusted R-squared value.

	(1)	(2)	(3)	(4)	(5)
Physical I/K		0.020***	0.020***		
		(10.69)	(11.78)		
Intangible I/K		0.019***	0.027***		
		(6.96)	(10.62)		
Total I/K				0.046***	0.036***
				(15.99)	(11.82)
Log MPK	−0.024***		−0.025***	−0.025***	−0.026***
	(−16.33)		(−19.04)	(−18.82)	(−19.19)
Log age					−0.010***
					(−17.30)
Ind × Year FE	x	x	x	x	x
R^2	0.040	0.029	0.051	0.049	0.052
N	197,725	197,486	197,486	197,725	197,725

MPK, a higher investment rate is linked to a greater probability of experiencing a jump. Consequently, firms that continue to invest in spite of having low MPK are more likely to encounter rapid growth.

Columns 2 and 3 of Table 1 highlight the importance of both physical and intangible investments as relevant predictors of jumps, with comparable magnitudes. As per Peters and Taylor (2017), we combine physical and intangible investments to derive the firm's total investment and total capital. The results in Column 4 indicate that total investment is able to capture the explanatory power of its individual physical and intangible components in predicting jumps. Lastly, in Column 5, we introduce firm age as an additional factor. The results demonstrate that younger firms have a higher probability of experiencing jumps, indicating that firm age plays a role in predicting booms. Notably, the predictive ability of high investment and low MPK is not overshadowed by firm age, highlighting their independent contributions in identifying firms likely to experience jumps.

The implications of the results presented in Table 1 become clearer when we examine two firms with identical and high investment rates but differing levels of current MPK. Consider a firm with high MPK, indicating that it already has high output relative to its capital. In this case, the firm's high investment rate can be justified by the potential benefits of having more capital to further enhance its output. On the other hand, we have a low MPK firm that seemingly has excess capital compared to its output, yet it still falls within the high investment group. This combination of characteristics, namely high investment and low MPK, poses a challenge when analyzing the firm solely based on its current observable characteristics. Nevertheless, they are more likely to experience rapid growth in the future, suggesting that their investment decisions are influenced by factors beyond current MPK. This indicates the presence of alternative channels or factors driving their investment behavior, possibly related to their expectations of future growth opportunities, and challenges the prevailing notion that a lower MPK among firms in the cross-section represents misallocated capital that could be more efficiently utilized by high MPK firms.

We next adopt a portfolio approach and analyze the characteristics of firms that are sorted based on both their total I/K and MPK. This approach allows us to investigate the significance and implications of MPK variation among firms with similar investment, and helps us understand the potential economic channels behind the high I/K-low MPK phenomenon. Accordingly, we sort firms into four portfolios based on their placement in below or above median groups for both I/K and MPK. To ensure that our findings are not driven by variations across industries, we conduct the sorting within each 2-digit SIC industry, allowing us to focus on within-industry comparisons. In a frictionless economy where firm productivity is static or completely exogenous, we would expect investment rate and MPK to be perfectly correlated. Consequently, all firms would fall into either the low I/K - low MPK ($I/K_1, MPK_1$) portfolio or the high I/K - high MPK ($I/K_2, MPK_2$) portfolio. However, Panel A of Table 2 reveals that 40% of Compustat firms are placed in off-diagonal portfolios.[5]

In line with the results of Table 1, we observe that firms in high I/K - low MPK ($I/K_2, MPK_1$) portfolio have a significantly higher probability of experiencing a jump than firms in other portfolios. Specifically, these firms have an annual jump probability of 3.7%, which is more than twice as high as the average jump probability of the entire sample. These firms are also younger, which is associated with a higher likelihood of experiencing a jump, as shown in Table 1. While 21% of all firms belong to the ($I/K_2, MPK_1$) portfolio, this proportion increases to 26% among young firms, defined as those firms that have been included in the Compustat sample for 10 years or less.

When examining the characteristics of the ($I/K_2, MPK_1$) portfolio over time, several intriguing patterns emerge. As shown in Panel A of Figure 1, the share of firms in this portfolio remains relatively stable throughout the sample period with minor fluctuations.[6] In contrast, the annual jump probability depicted in Panel B varies over time, reaching its peak in the late 1990s. Interestingly, these are also the years when firms in this group experienced their highest investment rate and had the lowest MPK relative to the median firms in their industry (Panel C). Moreover, the investment rate and MPK move in opposite directions, and jumps comove positively with the investment rate and negatively with MPK.

[5]Based on the evidence presented in Table 1, we have defined total capital as the sum of physical and intangible capital. Our I/K and MPK are also consistent with this definition. In Appendix Table A.1, we have conducted portfolio sorting based solely on I/K and MPK derived from physical capital. While the results exhibit similarities with those presented in Table 2 across most dimensions, we have observed that the distinctions between the ($I/K_2, MPK_1$) portfolio and the other portfolios are somewhat less pronounced both economically and statistically.

[6]Appendix Figure A.1 reproduces Figure 1 using only physical capital, demonstrating that the time series dynamics of the ($I/K_2, MPK_1$) portfolio's share, jump probability, investment, and MPK remain largely unchanged.

Table 2: Descriptive Statistics for Total I/K and MPK Sorted Portfolios

To construct the four portfolios, all firms are annually sorted into below and above median I/K and MPK groups. The resulting portfolio statistics are presented in columns 1 to 4, while columns 5 and 6 display the differences between firms in the $(I/K_2, MPK_1)$ group and the entire sample and their $t-$statistics. For each variable, the statistics are initially computed for all firms within each portfolio and then averaged across years. Appendix A provides a detailed explanation of variable definitions and sources. To account for industry differences, total I/K, log MPK, and log TFP are normalized by subtracting the median values of their respective 2-digit SIC industries, and the median SA index is normalized to be 1 each year to increase readability. Excess future stock returns $(r - r^f)$ are measured from July of year $t+1$ to June of year $t+2$. Most variables are reported as portfolio medians, except for TFP where the 90th percentile is also presented. Excess returns $(r - r^f)$ are calculated as value-weighted averages, while patent-based variables are reported as means due to the highly skewed nature of patenting activity.

	$(I/K_1, MPK_1)$	$(I/K_1, MPK_2)$	$(I/K_2, MPK_1)$	$(I/K_2, MPK_2)$	$(I/K_2, MPK_1)$–*All*	
					Difference	t–stat
Panel A: Portfolio properties						
N	1534.5	1052.5	1061.1	1494.3		
Total I/K (median, ind. adj.)	-0.059	-0.048	0.086	0.100		
Log MPK (median, ind. adj.)	-0.40	0.34	-0.35	0.44		
Portfolio share	0.30	0.20	0.21	0.29		
Portfolio share among young firms ($\leq$ 10 years)	0.22	0.15	0.26	0.37		
Age (median)	13.8	14.7	8.88	8.66	-2.15***	-3.49
Jump probability (%)	1.42	0.53	3.71	0.83	2.21***	7.86
Panel B: Innovative activity and product development						
Patents/K (mean)	10.4	7.12	28.6	18.1	12.8***	5.65
Patent value/K (mean)	24.0	20.0	93.9	90.9	36.7**	2.37
Patent Citations/K (mean)	300.8	145.4	1174.7	580.7	639.4***	4.25
Top 20% patents/K (mean)	2.18	1.07	7.67	3.95	4.04***	4.04
Top 10% patents/K (mean)	1.24	0.49	4.72	2.12	2.64***	3.77
Top 5% patents/K (mean)	0.73	0.25	2.86	1.16	1.65***	3.48
Exposure to Life1 stage (median)	0.22	0.18	0.29	0.25	0.061***	11.7
Panel C: Productivity, returns, financial constraints						
Log TFP (median, ind. adj.)	-0.062	-0.034	0.021	0.078	0.021***	4.52
Log TFP (90th pctile, ind. adj.)	0.34	0.33	0.49	0.58	0.047**	2.28
Log future TFP (5yr later, median, ind. adj.)	-0.021	-0.020	0.014	0.010	0.017***	3.65
Log future TFP (5yr later, 90th pctile, ind. adj.)	0.41	0.36	0.54	0.49	0.098***	4.29
Excess future stock returns (VW mean, annual, %)	8.09	8.26	8.10	8.37	0.22	0.053
SA index (median)	-0.10	-0.25	0.18	0.13	0.18***	8.70

These time series patterns are consistent with the cross-sectional results presented in Table 1: not only are firms with high investment and low MPK more likely to jump in the cross-section, but a larger fraction of these firms experience a jump when median investment is high and median MPK is low in the time series.

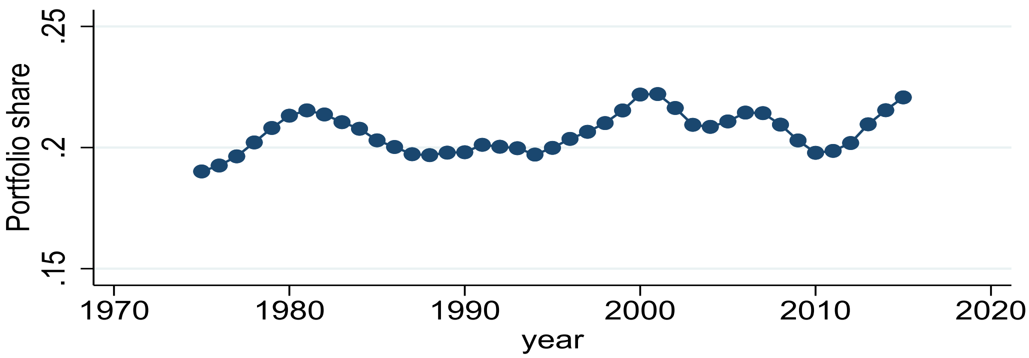
Jumps in firm performance can stem from various factors fueled by investment. These jumps are characterized by rapid growth, and they are not randomly distributed among firms but strongly associated with both investment and MPK. Examples of such factors are innovative activity and new product development, which often require sustained investment over a considerable period, with the anticipation of potential but uncertain future benefits.

To gain insights into the differences in firm jump propensity, we examine outcomes that capture innovative activity and new product development, as proposed in recent literature. To evaluate innovative quantity and quality, we employ multiple patent-based measures, such as traditional patent and citation counts, as well as patent values derived from stock market reactions to patent news developed by Kogan, Papanikolaou, Seru, and Stoffman (2017) and breakthrough patent measures that identify the most innovative and influential patents using textual analysis proposed by Kelly, Papanikolaou, Seru, and Taddy (2021). To ensure comparability of innovative intensity across firms of varying sizes, we scale all patent measures by the firm's total capital.

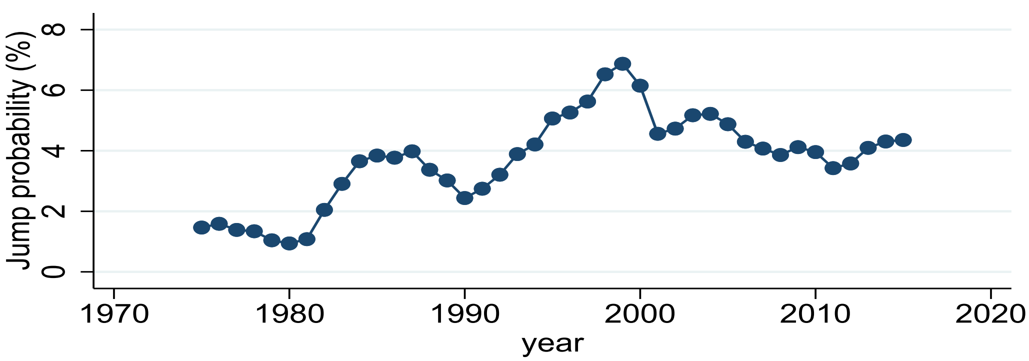
The average innovation metrics for the portfolios are presented in Panel B of Table 2. We find that firms in the $(I/K_2, MPK_1)$ portfolio exhibit higher patent issuance, with these patents being more valuable and receiving more citations. Notably, this portfolio stands out in terms of top (breakthrough) patent issuance and patent citation measures, surpassing the performance of all other portfolios. This finding is consistent with a higher jump propensity in performance. To measure new product development activity, we use firm exposure to Life1 (product innovation) stage, which is identified by textual analysis of 10-K files by Hoberg and Maksimovic (2022). As shown in Panel B of Table 2, firms in the $(I/K_2, MPK_1)$

Figure 1. $(I/K_2, MPK_1)$ Portfolio share and jump probability over 1975-2015

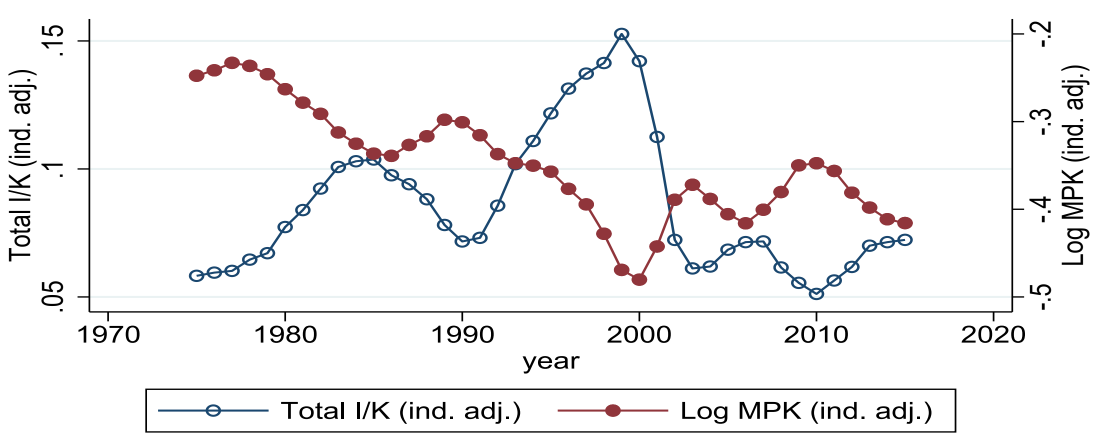
The figure illustrates the portfolio share, jump probability, and median investment rate and MPK for firms in the $(I/K_2, MPK_1)$ portfolio between 1975 and 2015. Panel A presents the portfolio shares, Panel B displays the jump probabilities, and Panel C shows the industry-adjusted median investment rate and log MPK. All variables are presented as 3-year moving averages.



(a) Share of Compustat firms placed in $(I/K_2, MPK_1)$ portfolio



(b) Annual jump probability of firms in $(I/K_2, MPK_1)$ portfolio



(c) Median I/K and MPK of firms in $(I/K_2, MPK_1)$ portfolio

portfolio exhibit significantly higher exposure to this stage of product development compared to the other portfolios. The Life1 stage is characterized as risky since firms need to acquire capacity before knowing the outcome of product development, which aligns with the notion that investment predicts future jumps.

We also compare current and future firm productivity (total factor productivity, TFP) across portfolios using TFP estimates from İmrohoroğlu and Tüzel (2014). Panel C Table 2 shows that firms in high I/K portfolios exhibit higher productivity levels than firms in low I/K portfolios. Additionally, among the two high investment portfolios, firms in the high MPK portfolio have higher median and 90th percentile productivity levels compared to firms in the $(I/K_2, MPK_1)$ portfolio. However, the pattern shifts in the subsequent years, as TFP of firms in the $(I/K_2, MPK_1)$ portfolio exceeds other portfolios five years later. More strikingly, this improvement in TFP for firms in the $(I/K_2, MPK_1)$ portfolio is particularly prominent among top performing firms in each portfolio (such as firms in the 90th percentile), indicating that currently high I/K - low MPK firms are likely to be in the right tail of TFP distribution in five years, experiencing substantial jumps in performance.

To gain further insight into the potential longer-term jump outcomes of high I/K and low MPK firms, which tend to be younger, we define "high-type" firms in the dataset as those that remained in the $(I/K_2,\ MPK_1)$ portfolio for at least 5 years within their first 10 years of inclusion in the Compustat sample. Of the firms that joined Compustat after 1975 and remained for at least 5 years, 12% are identified as high-type firms. Among these high-type firms, we find that 28% experienced at least one jump by 2015, which is more than twice the average life-time jump probability of the entire firm sample (13%). Many of these firms eventually become significant players in the economy. Specifically, more than 5% of high-type firms are eventually included in the S&P 500, compared to only 3.5% of the entire sample of firms.

A large body of research has highlighted the importance of risk premia in explaining the cross-sectional differences in investment behavior. As noted by David, Schmid, and Zeke (2022), it is the appropriately discounted marginal product of capital (MPK) that should be equalized across firms when capital is efficiently allocated, rather than the MPK itself. Therefore, it is essential to account for any differences in risk premia across portfolios. Panel C of Table 2 reports the value-weighted average excess stock returns for the portfolios in the year following the portfolio formation.[7] We find that the realized risk premia were remarkably similar across the portfolios in our sample period. This suggests that the dispersion in MPK and I/K were not driven by variations in the discount rates of the portfolios.

Recent literature has highlighted financial frictions as a potential source of misallocation (Midrigan and Xu, 2014; Moll, 2014; Whited and Zhao, 2021; Bau and Matray, 2023). To examine whether such frictions may explain the low investment despite high MPK and high investment despite low MPK that we observe in off-diagonal portfolios, we employ the SA index proposed by Hadlock and Pierce (2010), which uses firm age and size as key indicators of financial constraints. Our findings show that $(I/K_2, MPK_1)$ has the highest SA index among the four portfolios, indicating that it has the most financially constrained firms. In contrast, $(I/K_1, MPK_2)$ has the least financially constrained firms. Therefore, if financial constraints are indeed impeding investment, we would expect $(I/K_2, MPK_1)$ firms to invest even more in the absence of such constraints, leading to an even wider gap between investment and MPK. Conversely, the low investment of $(I/K_1, MPK_2)$ firms is unlikely to be due to financial constraints. Hence, the observed "misallocation" in the off-diagonal portfolios cannot be attributed to the financial constraints faced by these firms. In summary, our findings suggest that removing financial constraints would not necessarily redirect more capital exclusively to high MPK firms, but also potentially to firms in the $(I/K_2, MPK_1)$

[7]Following the standard convention, we match CRSP stock return data from July of year $t+1$ to June of year $t+2$ with accounting information for the fiscal year that ended in year t, as in Fama and French (1992) and Fama and French (1993).

portfolio.

The evidence indicating that high-type (young, high I/K - low MPK) firms have the potential to make substantial leaps in innovation, productivity, and sales contradicts the standard neoclassical firm model, which primarily relies on marginal product of capital (MPK) for capital allocation. Instead, this evidence supports features of endogenous growth models, which suggest that firms pursue investments to innovate, enhance production processes, and introduce new products to achieve higher growth. In the following section, we present a simple quantitative model of an economy that extends the neoclassical framework with a jump augmented productivity process. This addition incorporates aspects of the endogenous growth model in a reduced-form and concise manner. Importantly, the model disrupts the near perfect correlation between I/K and MPK, and gives rise to a nuanced and heterogeneous relationship between the two. When applied to data from Compustat firms, the model produces significant variation in MPK, allowing for the analysis of counterfactual experiments with different firm policies and their impact on MPK and productivity outcomes.

3 Model

In this section, we introduce a simple model that differentiates between firms with and without the prospects of a large positive jump in the productivity of capital. Let i denote the index of firms. Every firm uses capital $K_{i,t}$ at time t as the single productive factor whose law of motion is given by

$$K_{i,t+1} = (1 - \delta)K_{i,t} + I_{i,t}, \tag{1}$$

where δ is the depreciation rate and $I_{i,t}$ is investment. In line with our empirical analysis, capital in the model encompasses both intangible capital such as R&D, branding, and new product development, and physical capital. Investment is subject to standard quadratic adjustment costs on net investment and the total cost of investment is given by $I_{i,t} + \frac{1}{2}c\left(\frac{I_{i,t}}{K_{i,t}} - \delta\right)K_{i,t}$.

Following Klette and Kortum (2004) and Acemoğlu, Akçiğit, Alp, Bloom, and Kerr (2018), we assume that firms differ in their potential for experiencing productivity jumps. Upon entering the economy, a firm's type θ_i, either high or low, is randomly assigned, and each newly established firm has an identical probability p of being categorized as high-type:

$$\Pr(\theta_i = \theta^h) = p \text{ and } \Pr(\theta_i = \theta^l) = 1 - p, \tag{2}$$

where $p \in (0, 1)$.

Firm i produces output $Y_{i,t}$ based on the technology given by

$$Y_{i,t} = Z_{i,t}^{\alpha} K_{i,t}^{1-\alpha}, \tag{3}$$

where $Z_{i,t}$ denotes productivity. High-type and low-type firms differ in their stochastic productivity process. Log productivity $z_{i,t}$ of a high-type firm follows

$$z_{i,t+1} = z_{i,t} + \epsilon_{i,t+1} + J_{i,t+1}, \tag{4}$$

where ϵ_i is a Gaussian shock with mean zero and volatility σ, and J_i represents a jump shock.[8] In each period, jumps are characterized by

$$\Pr(J_{i,t+1} = \zeta) = \lambda_{i,t} \text{ and } \Pr(J_{i,t+1} = 0) = 1 - \lambda_{i,t}, \tag{5}$$

[8] Our formulation for firm productivity follows a similar structure to the cash flow formulation used by Andrei, Mann, and Moyen (2019), which includes a Gaussian component, as well as an innovation-driven jump component.

where $\lambda_{i,t}$ is the probability of a jump and $\zeta > 0$ is the time-invariant jump size. Low-type firms' productivity does not feature jumps and follows $z_{i,t+1} = z_{i,t} + \epsilon_{i,t+1}$.

The jumps in high-type firms' productivity capture infrequent and large upward moves in firm sales and productivity documented in Section 2. Motivated by the significantly higher empirical jump propensity of high I/K - low MPK firms, we allow the probability of jumps to vary with capital accumulation.[9] In particular, we parameterize jump probability $\lambda_{i,t}$ as

$$\lambda_{i,t} = \lambda_0 \left(\frac{k_{i,t}}{k^{ss}} \right)^{\iota}, \tag{6}$$

where $k_{i,t} = K_{i,t}/Z_{i,t}$ and k^{ss} is the steady-state value of $k_{i,t}$. As a result, λ_0 controls the level of the jump probability and $\iota > 0$ determines its curvature with respect to the firm capital level. Firms can accumulate more capital relative to their current productivity to increase the likelihood of a jump, giving rise to a new determinant of capital investment. In other words, firms invest not only for current production opportunities, but also to increase their expected future production. For instance, to achieve potentially rapid growth, firms need to invest in building up brand capital, developing technology, as well as obtaining market share to become potential "superstar" firms, which are characterized as the most productive firms of an industry in Autor, Dorn, Katz, Patterson, and Van Reenen (2020). In that sense, our broad definition of capital encompasses factors that not only drive a firm's current production but also its future potential.

Similarly to Acemoğlu, Akçiğit, Alp, Bloom, and Kerr (2018), we assume that high-type firms transition to low-type at the exogenous flow rate μ, and low-type is an absorbing state. Furthermore, each firm is subject to an exogenous destruction rate φ. In case of destruction, firm value declines to zero and the firm exits the economy.

[9] This approach is consistent with the modeling of innovation jumps in the literature. For instance, Klette and Kortum (2004) propose that a firm's innovation rate is determined by both its R&D investment and its knowledge capital. Andrei, Mann, and Moyen (2019) make a similar assumption.

Given the structure of firms described so far and assuming that firms discount cash flows at rate R, high-type firms solve the following value maximization problem:

$$\begin{aligned}V^h(K_{i,t}, Z_{i,t}) = \max_{I_{i,t}} &\left(Z_{i,t}^{\alpha} K_{i,t}^{1-\alpha} - I_{i,t} - \frac{1}{2} c \left(\frac{I_{i,t}}{K_{i,t}} - \delta \right)^2 K_{i,t} \right) \\ &+ \frac{1}{R}(1-\varphi) \left((1-\mu)\mathbb{E}_t \left[V^h(K_{i,t+1}, Z_{i,t+1}) \right] + \mu \mathbb{E}_t \left[V^l(K_{i,t+1}, Z_{i,t+1}) \right] \right),\end{aligned} \tag{7}$$

and low-type firms solve

$$\begin{aligned}V^l(K_{i,t}, Z_{i,t}) = \max_{I_{i,t}} &\left(Z_{i,t}^{\alpha} K_{i,t}^{1-\alpha} - I_{i,t} - \frac{1}{2} c \left(\frac{I_{i,t}}{K_{i,t}} - \delta \right)^2 K_{i,t} \right) \\ &+ \frac{1}{R}(1-\varphi) \mathbb{E}_t \left[V^l(K_{i,t+1}, Z_{i,t+1}) \right].\end{aligned} \tag{8}$$

To obtain optimal policies, we utilize value function iteration by normalizing the firm value functions V^h and V^l in equations (7) and (8) with respect to productivity Z. See Appendix B for details. In the quantitative implementation of the model described in Section 4, we make the assumption that all shocks $\epsilon_{i,t}$ and $J_{i,t}$ are independent across firms and time, and we focus solely on the cross-sectional implications by abstracting from any common variations in these shocks. Additionally, we consider a fixed number of firms, meaning that when a firm is hit by an exit shock (with probability φ), it is replaced by a new firm with a high-type probability of p.

4 Quantitative analysis

In this section, we discuss our estimation strategy and examine the quantitative implications of our model, considering both targeted and non-targeted moments. Additionally, we perform counterfactual exercises utilizing the estimated model to illustrate the role of jumps within the overall framework and the impact of expected jumps on investment policy, thereby influencing both the cross-sectional distribution of firms and aggregate productivity. Lastly, we provide empirical evidence that supports the predictions derived from comparing the baseline model to the counterfactual model.

4.1 Model estimation

We estimate the model parameters using data from Compustat firms. In order to streamline the computation process, we predetermined a subset of parameters outside of the estimation. First, we set the depreciation rate δ to 15%, which falls within the commonly used range for physical and intangible capital.[10] Second, we set the discount rate $1/R$ to 0.91 for all firms, which corresponds to the (inverse of the) value-weighted average real stock return for the firms in our sample.[11] Third, we set the curvature parameter of the production function α to 0.35, in accordance with commonly used values in the literature (David, Schmid, and Zeke, 2022). Finally, we determined the exit probability directly from the data and set φ to 2.9%.[12]

We estimate the the remaining 7 parameters jointly using the simulated method of moments (SMM). Let $\theta = \{c, \sigma, \lambda_0, \iota, \zeta, \mu, p\}$ denote the set of seven parameters to be estimated. The SMM approach minimizes the weighted quadratic distance between a vector of moments

[10]In the literature, the depreciation rate for physical capital typically ranges between 8-10% (Jones and Tüzel, 2013), while for intangible capital, it is around 20% (Peters and Taylor, 2017). In our sample, the median ratio of intangible capital to physical capital is 1.1. To determine the depreciation rate for total capital, we computed the weighted average of the depreciation rates for physical and intangible capital. Our results are not sensitive to the choice of depreciation rate, except for the investment rate.

[11]As shown in Table 2, the realized risk premia were remarkably similar across the portfolios formed based on I/K and MPK during our sample period.

[12]We observed that 6.9% of firms in the Compustat sample exited every year. We distinguish between exits due to mergers and acquisitions (which were uniformly 4% across our portfolios) and other exits, which were classified as firm failures. Further details on the classification of Compustat exits can be found in Appendix A.3. Assuming that in the case of mergers, investors were paid the current value of the firm, mergers do not affect the firm's optimization problem. However, we took the merger rate into account in the simulations of the model.

in the data Ψ^d and model-simulated data $\Psi^m(\theta)$:

$$\arg\min_{\theta} g(\theta)' W g(\theta), \tag{9}$$

where $g(\theta) = \Psi^d - \Psi^m(\theta)$. In order to ensure that our estimated structural parameters are economically meaningful, we carefully select moments from the data that are closely related to them. As a result, we identify three sets of moment conditions: the first set consists of general moments that capture the volatility of investment and output outcomes. The second set includes moments that are specifically linked to the jump mechanism that we empirically illustrate in Section 2 and discuss in Section 3. Finally, the third set of moments pertains to the portfolio characterized by high investment and low MPK, which represents the primary focus of our paper.

Table 3 lists the target moments in our indirect inference procedure. Details on the empirical computations can be found in Appendix A.2. We formulate the model at an annual frequency, consistent with our empirical work, and Table 4 displays the calibrated and estimated parameter values.

Although all of the model parameters contribute to all of the moments, some of these moments are particularly informative for identifying certain parameters. The volatility of Gaussian shocks σ contributes to increased dispersion in both output and investment.[13] Conversely, adjustment costs limit large capital adjustments, resulting in a substantial reduction in investment volatility, with a relatively smaller impact on sales volatility. By jointly targeting investment and output dispersion, we can accurately estimate these parameters. Additionally, the model exhibits distinctive investment dynamics, with young, primarily

[13] Instead of using the standard deviation, which can be sensitive to outliers and winsorization of firm-level data, we utilize the interquartile range (IQR) to measure investment and output dispersion. Unlike the standard deviation, the IQR is not influenced by extreme values, making it a more robust measure of dispersion.

Table 3: Moment Conditions

This table presents the target empirical moments and model-generated moments obtained from the estimation of our baseline and counterfactual models. The empirical moments, presented in column 1, are computed using Compustat data from the period 1975-2015. For more details, please refer to Appendix Section A.2. The parameter estimates for the benchmark model can be found in Table 4. The counterfactual model, as discussed in Section 4.2.2, adopts the same parameter estimates as the baseline model, except that firms' investment policy ignores the possibility of jumps.

	Data	Baseline	Counterfactual
Panel A: General moments			
IQR of I/K among young firms	0.247	0.238	0.087
IQR of I/K among mature firms	0.094	0.118	0.103
Nonnegative investment share	0.988	0.944	0.978
IQR of sales growth	0.244	0.187	0.157
Panel B: Moments related to jump realizations			
Median sales jump size	2.930	2.763	2.957
Median log MPK jump size	0.710	0.632	0.609
Median jump age	6.000	6.000	5.000
Panel C: Moments for the $(I/K_2, MPK_1)$ portfolio			
Portfolio share	0.206	0.180	0.000
Jump probability	0.038	0.034	0.000
I/K (ind. adj.)	0.086	0.082	0.000
Portfolio share among young firms	0.258	0.275	0.000

high-type firms experiencing both Gaussian and jump shocks, while old, primarily low-type firms are only exposed to Gaussian shocks. We separately consider investment dispersion for young and old firms and incorporate the nonnegative investment rate, as in Tüzel (2010), in the targeted moments, to ensure that the parameters influencing investment dynamics are appropriately estimated.

The primary mechanism in our model hinges on the impact of anticipated rare jumps on firm investment. To capture this mechanism, we include key moments on the measurement of realized jumps in our moment conditions. In our model, the magnitude of jump realizations, conditional on a returns-to-scale parameter α, is primarily determined by the jump size parameter ζ. We include the measured jump sizes for sales and log MPK in the moment

Table 4: Parameter Estimates

This table provides the parameters resulting from the estimation of the baseline model. Panel A displays the calibrated parameters, while Panel B presents the estimated parameters, with the target and model-generated moments shown in Table 3. The standard errors for the parameters are presented in parentheses.

Panel A: Calibrated parameters	
Capital depreciation rate, δ	0.15
Discount rate, $1/R$	0.91
Production function curvature, α	0.35
Exit probability, φ	0.029
Panel B: Estimated parameters	
Gaussian shock volatility, σ	0.322
	(0.056)
Adjustment cost parameter, c	2.930
	(0.801)
Jump probability level, λ_0	0.028
	(0.007)
Jump probability curvature, ι	0.301
	(0.222)
Jump size, ζ	2.334
	(0.315)
Type switching probability, μ	0.082
	(0.022)
Probability of being born high-type, p	0.960
	(0.148)

conditions (Panel B of Table 3). To measure these jump sizes in simulations, we follow the same procedure employed in the data, as outlined in Section 2. Additionally, we include the median jump age in our moment conditions, which helps discipline the level of jump probability λ_0 as well as the type-switching probability μ, given the absence of jumps upon transitioning to low-type.

Our model emphasizes the unique implications of rare jumps for the cross-sectional distribution of firms. High-type firms invest more than expected given their current productivity because they anticipate rapid growth in the future, which they can influence through investment. This results in a large number of firms with relatively high levels of capital compared to their sales (low MPK firms) but also high growth rates (high investment). In order to estimate parameters associated with jump probability, λ_0 and ι, we utilize moment conditions for this particular group during the estimation process, which are presented in Panel C of Table 3. The fraction of this group among all firms is influenced by changes in investment policy due to the anticipation of jumps, which in turn, is determined by the jump probability. Empirically, we observe that the proportion of high investment–low MPK firms among young firms (within ten years after establishment) is higher compared to the proportion among all firms (25.8% versus 20.6%). We target both of these fractions in our estimation, which align with the notion that high-type firms transition to low-type firms over time. The type-switching probability, denoted as μ, plays a crucial role in determining the expected duration of being a high-type firm and directly influences the present value of investing an additional unit of capital for such firms. This parameter exhibits significance in determining both the average jump age and the average investment rate of high-type firms in comparison to low-type firms. Lastly, we estimate the probability that a new firm is born as high-type, p, which determines the share of high-type firms in the economy, along with μ.

Table 3 demonstrates that our baseline model provides a reasonably good fit to the data, despite targeting 11 moments with only 7 parameters, resulting in an overidentified system. In particular, the model successfully generates substantial dispersion in sales growth and investment rates, with higher investment dispersion among young firms, consistent with empirical observations. Importantly, our model also replicates large movements in sales and MPK that qualify as jumps, utilizing our empirical jump identification method, which we apply to the model simulations as well.

Panel B of Table 4 reports the parameter estimates underlying the simulated moments in Table 3. General moments capturing investment and sales growth dispersion are key for estimating the adjustment cost parameter c and Gaussian shock volatility σ. While a higher volatility generates both higher sales growth and higher investment dispersion, high adjustment costs in the estimation prevent the investment dispersion from being counterfactually high while having a limited effect on sales growth. Furthermore, the nonnegative investment share decreases with volatility and increases with the adjustment cost parameter.

Moments related to jumps and the $(I/K_2, MPK_1)$ portfolio in Panels B and C of Table 4 play a crucial role in shaping the estimation of other parameters. Specifically, the estimation selects a jump size of 2.33 to match the observed jumps characterized by a doubling of sales and a 50% increase in MPK over a two-year period. Additionally, the jump probability of the $(I/K_2, MPK_1)$ portfolio aids in identifying both the level and the curvature of the jump probability. The estimation also reveals an 8.22% switching probability from high-type to low-type firms and a significant fraction of new firms being born as high-type (96.0%). As conjectured, the moment conditions effectively identify the estimated parameters, resulting overall in low standard errors.

Finally, the $(I/K_2, MPK_1)$ portfolio is constructed in the model to mirror the data. In the model, 18% of simulated firms belong to this portfolio (compared to 20% in the data). These firms exhibit an average annual jump probability of 3.4% in the model (3.8% in the data). Notably, the median jump probability in this portfolio significantly exceeds the median probability among all firms, both in the model (1.1%) and in the data (1.5%), even though it is not a directly targeted moment in the estimation. Also consistent with the data, the proportion of the $(I/K_2, MPK_1)$ portfolio is higher among young firms. Moreover, the median investment rate within this portfolio surpasses the median among all firms by approximately 8.2 percentage points in the model (8.6pp in the data).

In sum, the model rationalizes the presence of firms with above-median investment rates despite below-median MPKs, which may initially appear to be misallocating productive capital. The characteristics of the $(I/K_2, MPK_1)$ portfolio guide our mechanism that generates this portfolio: the anticipation of large positive moves in their future productivity drives high investment rates despite their low current productivity levels. As a result, we argue that the high capital allocation to low MPK firms does not necessarily indicate a misallocation of resources.

4.2 Inspecting the mechanism

In this section, we delve deeper into our model's mechanism and address two fundamental questions. First, can a model with conventional Gaussian shocks match the targeted empirical facts regarding jumps without using Poisson shocks as in our baseline model? Second, what is the role of investment policy in response to anticipated jumps? Specifically, what can we infer from a counterfactual scenario where firms do not invest and grow in anticipation of productivity jumps?

By addressing these questions, we aim to gain a better understanding of the mechanisms underlying our model and shed light on the role and consequences of investment policies tied to the anticipation of jumps.

4.2.1 Model implications in the absence of jumps

To answer the first question, we perform two estimations using a modified version of the model that completely eliminates jumps and keeps all other features the same as in Section 3. This model is essentially a standard neoclassical model with homogeneous firms, random walk productivity, and quadratic adjustment costs. Consequently, the only parameters to estimate are c and σ, as all other estimated parameters in the baseline model relate to jumps and firm types.

Table 5 presents the simulated moments obtained from the estimation of the no-jump model. First, we target the moments in Panel A related to investment and sales growth

Table 5: Moments in a Model with No Jumps

This table displays the target empirical moments and model-generated moments obtained from the estimation of a model without jumps, as discussed in Section 4.2.1. The empirical moments, presented in column 1, are computed using Compustat data from the period 1975-2015. For more detailed information, please refer to Appendix Section A.2. Column 2 presents the model-generated moments resulting from an estimation that focuses solely on matching the moments in Panel A. Panel B presents the moments when both Panels A and B are targeted. Parameter estimates for both estimations can be found in Table 6.

	Data	Targeting Panel A	Targeting Panel A & B
Panel A: General moments			
IQR of I/K among young firms	0.247	0.108	0.095
IQR of I/K among mature firms	0.094	0.107	0.091
Nonnegative investment share	0.988	0.801	0.822
IQR of sales growth	0.244	0.234	0.244
Panel B: Moments related to jump realizations			
Median sales jump size	2.930	2.131	2.194
Median log MPK jump size	0.710	0.432	0.445
Median jump age	6.000	8.000	7.000
Jump probability (in %)	1.499	0.008	0.012
Panel C: Moments for the $(I/K_2, MPK_1)$ portfolio			
Portfolio share	0.206	0.000	0.000
Jump probability	0.038	0.000	0.000
I/K (ind. adj.)	0.086	0.000	0.000
Portfolio share among young firms	0.258	0.000	0.000

dispersion and observe that the estimation selects higher values for c and σ compared to the baseline model (Table 6). The absence of jumps in this model requires higher volatility to match the dispersion of sales growth. However, this also necessitates higher adjustment costs to prevent investment dispersion and the negative investment share from being counterfactually high. While this model successfully matches the dispersion among mature firms, it fails to generate additional investment dispersion among young firms due to the absence of young high-type firms. Panel B of Table 5 reveals an interesting finding: there are some observations of sales growth and log MPK growth that are classified as jumps in the no-jump model. However, the probability of these large moves is very small in this model (0.01%)

Table 6: Parameter Estimates in a Model with No Jumps

This table presents the parameters obtained from the estimation of the no-jump model, as discussed in Section 4.2.1. The corresponding target and model-generated moments can be found in Table 5. Column 1 displays the parameters resulting from an estimation that only targets the moments in Panel A of Table 5. Column 2 presents the parameters when both Panels A and B are targeted. The standard errors for the parameters are presented in parentheses.

	Targeting Panel A	Targeting Panel A & B
Gaussian shock volatility, σ	0.513	0.537
	(0.006)	(0.004)
Adjustment cost parameter, c	3.357	3.995
	(0.411)	(0.742)

compared to the data (1.50%), despite their magnitudes conditional on realization being similar to the data. Furthermore, in this model, the $(I/K_2, MPK_1)$ portfolio is completely empty. This is because in the absence of jumps, there is a counterfactually tight positive relationship between investment and MPK in the cross-section of firms. Consequently, MPK resembles the endogenous state variable that determines investment for all firms, and low MPK directly implies low investment.

The observation that the no-jump model can generate a small number of large moves similar to the data motivates our next estimation, which targets both the general moments in Panel A and the jump-related moments in Panel B of Table 5. However, this estimation yields similar values for c and σ as shown in Table 6. The reason behind this outcome is that increasing the Gaussian volatility to improve the jump statistics leads to excessively high levels of investment and sales growth dispersion, which is not favored by the objective function of the estimation. Consequently, the targeted jump probability only increases from 0.008% to 0.012%, still very far from the data value of 1.5%. Similar to the case where jumps are not targeted, there are no firms in the $(I/K_2, MPK_1)$ portfolio, and the realized jumps in Panel B are solely the result of large random realizations of the Gaussian shocks.

4.2.2 Model implications without "investing in misallocation"

To address the second question posed in Section 4.2, we conduct a counterfactual analysis using our baseline model. In this counterfactual scenario, jumps in productivity are still present in the data-generating process, but firms' investment policies ignore the potential of future jumps. That is, high-type firms do not engage in "investing in misallocation," meaning they do not invest more than a low-type firm with the same productivity and capital, even though they have a positive probability of experiencing a jump and additional capital could potentially increase their likelihood of jumping. The only distinction between low-type and high-type firms in this counterfactual scenario arises from the realized jumps, which are not anticipated. For this counterfactual exercise, we maintain the same model parameters as in the baseline model.

Table 3 presents a comparison of the simulated moments in the baseline model and the counterfactual model. The results show that the counterfactual model fails to generate higher investment dispersion for young firms (Panel A), indicating that the difference in investment dispersion between young and mature firms can be attributed to high-type firms' heightened investment in anticipation of jumps. The reduction in sales growth dispersion is also observed, although not to the same extent as the decline in young firms' investment dispersion. Despite these differences, the counterfactual model simulations still exhibit jumps, and their magnitudes closely resemble those observed in the data (Panel B). However, the unconditional probability of jumps is significantly lower, at 0.65% compared to 1.1% in the baseline model. This discrepancy arises from the jump specification in equation (6). In the counterfactual scenario, where firms ignore jumps in their investment policies, they also overlook the heightened jump probability associated with upward deviations of capital from its steady-state value. Consequently, firms do not invest to increase their jump probability, leading to an overall decline in the average jump probability.

Panel C of Table 3 illustrates that in the counterfactual model, the joint distribution of investment and MPK leads to the $(I/K_2, MPK_1)$ portfolio share becoming zero. When firms ignore jumps in their investment policies, investment is solely determined by the current MPK, thereby eliminating the occurrence of high investment despite a low MPK. Consequently, our model predicts that this portfolio, which constitutes 21% of Compustat firms, would be absent when the mechanism in our model concerning the impact of anticipated jumps on investment is shut down, even though jump realizations still form part of the data-generating process.

We also examine the relationship between predicted jumps, investment, and MPK in both the baseline and alternative models. The results of this comparison can be found in Table 7. In the data, we observe that investment positively predicts jumps while controlling for MPK, indicating that higher levels of investment are associated with a greater likelihood of jumps. Conversely, MPK negatively predicts jumps while controlling for investment, suggesting that lower MPK values are associated with a higher probability of jumps. Importantly, the baseline model successfully captures this pattern.

The core mechanism of the baseline model centers around this relationship. For a given level of investment across firms, a lower MPK is associated with a higher probability of a jump. This is because high-type firms, despite experiencing a low current MPK, are incentivized to invest due to their higher expected growth prospects. However, this mechanism is absent in the counterfactual model. Consequently, the predicted coefficients on investment and MPK in the counterfactual model have the opposite sign compared to the observed data.[14]

The last two columns of Table 7 present the regression results from the no-jump model. In these models, we observe that neither investment nor MPK predict jumps. This is because

[14] In the counterfactual model, high Gaussian shocks lead to a higher investment rate but lower K/Z, resulting in a lower jump probability. Furthermore, in this model, investment and MPK exhibit near-perfect correlation, which introduces a multicollinearity problem in the regression.

Table 7: Predicting Jumps in the Model

This table presents coefficient estimates obtained from linear probability models analyzing realized jumps using both empirical data and model-simulated data. The dependent variable is the jump dummy, which takes a value of 1 when a jump occurs from the current period to the next period. Jumps are defined as cases where a company's sales double, accompanied by a minimum 50% increase in its MPK. Column 1 reproduces Column 4 of Table 1, which contains coefficient estimates based on empirical data. Columns 2 to 5 correspond to coefficient estimates obtained from simulations of the Baseline model, the counterfactual model, and two versions of the no-jump model, respectively. Point estimates for the coefficients are simulation averages, while the confidence intervals (presented in parentheses) are constructed from the 5th and 95th percentiles of the simulated distribution of each coefficient.

	Data	Baseline	Counterfactual	No Jump Targeting A	No Jump Targeting A & B
Total I/K	0.046	0.076	-0.128	0.003	0.010
		[0.056; 0.097]	[-0.218; -0.029]	[0.000; 0.016]	[-0.001; 0.051]
Log MPK	-0.026	-0.046	0.038	-0.000	-0.001
		[-0.060; -0.032]	[0.005; 0.068]	[-0.002; 0.000]	[-0.007; 0.001]

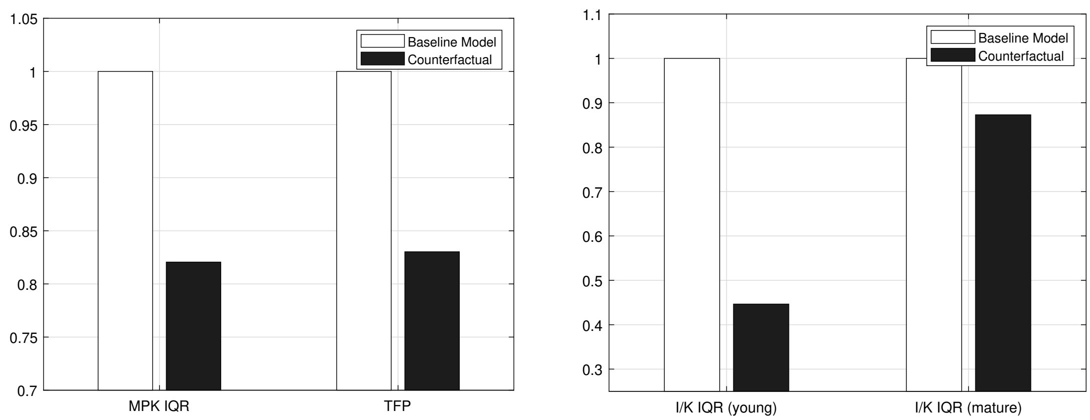
the occurrence of jumps cannot be predicted; it is solely driven by random realizations of large Gaussian shocks.

The cross-sectional dispersion in realized MPK is a commonly used metric for misallocation in the literature, where firms with low MPK are deemed to have too much capital relative to their more productive high MPK counterparts. We, therefore, investigate the extent to which cross-sectional dispersion in MPKs would be reduced in the counterfactual scenario where firms cease "investing in misallocation" compared to the baseline model. Figure 2 illustrates that the interquartile range of MPK is 18% lower in the counterfactual model. This reduction indicates a convergence of MPK values among firms. Additionally, the investment rates of firms also exhibit convergence, leading to a lower cross-sectional dispersion, particularly among young firms.

The 18% reduction in overall MPK dispersion may initially appear modest, but this figure fails to capture the heterogeneous effects on the cross-section of firms. A more nuanced understanding of this reduction emerges when examining MPK dispersion within portfolios

Figure 2. Dispersion and Aggregate Productivity in the Model

The figure illustrates MPK dispersion, aggregate productivity, and investment rate dispersion for both young and mature firms in the baseline and counterfactual models. In order to facilitate presentation and comparisons, the values in the baseline model are normalized to 1. The left panel displays the interquartile range (IQR) for MPK and the aggregate productivity level. The aggregate productivity level is computed as a Solow residual, obtained by subtracting $(1-\alpha)$ times the total capital from the total output produced by all firms. On the right panel, the investment rate IQR is displayed separately for young and mature firms.



categorized as above- and below-median I/K, as illustrated in Table 8. In the data, we observe significant MPK variation across portfolios independently sorted based on MPK within high and low investment groups. Notably, the baseline model generates approximately half of the dispersion in MPK observed in the data.[15] Interestingly, this variation is entirely eliminated in the counterfactual model due to the near-perfect correlation between investment and MPK. Therefore, MPK dispersion is not reduced randomly when jumps are ignored in investment policy. The unique feature of our model implies that it is the MPK dispersion

[15]Both in the data and the model, we measure MPK dispersions using realized MPK values, which is consistent with the existing literature. However, it is important to highlight that the median MPKs of portfolios in the model display a high degree of persistence, particularly for the off-diagonal portfolios. Consequently, the dispersion in both realized MPK and the expected MPK one period ahead across portfolios shows a similar magnitude, indicating that MPK dispersion is not primarily driven by realized shocks. So, why do firms in the $(I/K_2, MPK_1)$ portfolio continue to make substantial investments despite the current low MPK and the expectation of similarly low MPK in the next period? The answer lies in the endogenous growth feature of the model, where a high-type firm benefits from a considerably high present value of one unit of investment. This is attributed to the cumulative effect of investment on increasing the probability of future jumps beyond the next period, as well as the lasting impact of such jumps on future output. Consequently, capital is expected to remain elevated relative to productivity, even though the likelihood of a jump occurring in any given period may be relatively small.

Table 8: Model-Generated MPK Dispersion across Portfolios

This table compares the median MPK for I/K and MPK Sorted Portfolios using both empirical data and model-simulated data. Column 1 displays the median industry-adjusted MPK values for the portfolios, along with the MPK dispersion for the low and high investment groups. Columns 2 to 5 present the same statistics obtained from simulations of different models. Column 2 represents the Baseline model, Column 3 represents the counterfactual model, and Columns 4 and 5 represent two versions of the no-jump model.

	Data	Baseline	Counterfactual	No Jump Targeting A	No Jump Targeting A & B
$(I/K_1, MPK_1)$	−0.40	−0.24	−0.16	−0.28	−0.31
$(I/K_1, MPK_2)$	0.34	0.11	n/a	n/a	n/a
Difference	0.77	0.35	n/a	n/a	n/a
$(I/K_2, MPK_1)$	−0.35	−0.16	n/a	n/a	n/a
$(I/K_2, MPK_2)$	0.44	0.26	0.15	0.24	0.26
Difference	0.89	0.42	n/a	n/a	n/a

associated with the misalignment of investment and MPK in the cross-section that is eliminated in the counterfactual case. Similarly, while the no-jump model generates substantial overall MPK dispersion, as presented in the last two columns of Table 8, it also completely misses the mark in capturing the MPK dispersion within high and low investment firms.

What does our model reveal about the aggregate impact of eliminating our mechanism? Specifically, does the reduction in MPK dispersion in the counterfactual scenario have a positive or negative impact on overall efficiency? To address this question, we compute the aggregate Total Factor Productivity (TFP) in both the baseline and counterfactual models.[16] Figure 2 illustrates a striking 17% reduction in aggregate TFP when firms cease "investing in misallocation" and the associated reductions in MPK and investment dispersions take place.

In the baseline model, there are two opposing forces that influence TFP. On one hand, high-type firms invest in anticipation of jumps, resulting in higher capital relative to their current productivity, which suggests lower TFP. On the other hand, the infrequent realization

[16] We compute the aggregate TFP in the model as a Solow residual, obtained by subtracting $(1-\alpha)$ times total capital from the total output produced by all firms.

of anticipated jumps leads to substantial increases in productivity when they do occur, ultimately contributing to an overall rise in TFP. According to our model, higher investment driven by jump anticipation is justified by the expected gains from jumps, therefore such investment is expected to enhance overall productivity in the economy. In this sense, our mechanism generates favorable cross-sectional dispersion in MPK and investment, aligning with our interpretation of jumps as sources of innovation or other activities that foster future growth, despite their limited immediate impact on output. Therefore, a reduction in this beneficial dispersion implies diminished productivity in the economy, even though it stems from avoiding temporarily low-productivity firms whose investments may superficially appear as misallocation. In Section 4.3, we present empirical evidence that supports the notion that "investments in misallocation" have a net positive effect on aggregate productivity.

It is worth noting that our mechanism is distinct from the theme in the literature that relates misallocation to distortions such as adjustment costs or financial constraints (e.g., Hsieh and Klenow, 2009). In this literature, financially constrained firms cannot invest as much as they efficiently should, leading to persistent underinvestment and high MPK. Removing financial constraints results in increased allocation of capital to high MPK firms, reduced cross-sectional dispersion in MPK, higher output due to increased capital allocation to more productive firms, and generally does not affect individual firm productivity (Bau and Matray, 2023). Similarly, investment frictions typically lead to sluggish adjustment in capital, favoring the allocation of more capital to high MPK firms when frictions are eliminated (David and Venkateswaran, 2019).

Our mechanism complements the conventional view of misallocation by pointing out that allocating more resources to low MPK firms is not necessarily inefficient, considering their future prospects in terms of productivity, innovation, and growth. Therefore, our results suggest exercising caution when interpreting increased capital allocation to high MPK firms as the only efficient outcome, such as when financing constraints are lifted. Indeed, our mechanism suggests that allocating capital to high-type firms increases overall productivity while also allowing for the presence of low MPK firms. In summary, the patterns identified in our work and our model mechanism caution against labeling capital investment in low MPK firms as misallocation relative to high MPK firms.

4.3 Implications for firm growth and aggregate productivity

The analysis presented in Figure 2 highlights the potential for lower aggregate productivity, despite reduced MPK dispersion, if firms cease investing in anticipation of jumps. However, our simple framework assumes independence among firms and thus overlooks potential spillover effects. In reality, there may exist both positive and negative spillovers resulting from firms' investments and outcomes, which can have diverse impacts on other firms. As a result, the overall impact of our proposed channel, which clarifies the factors driving high investment among low MPK firms, on aggregate outcomes remains uncertain. For instance, investments in innovation can push the technology frontier, generating benefits for all firms. Conversely, investments aimed at expanding market share might adversely affect the growth prospects of other firms, or technologies developed by one firm could render the technologies of rival firms obsolete.

To empirically investigate this question, we explore the predictive relationship between firms' future growth and their current own investment, as well as the investment of their competitors. To this end, we employ the following regression model:

$$\log \frac{Y_{i,t+5}}{Y_{i,t}} = \alpha_0 + \alpha_1 \; I/K_{i,t} + \sum_{p=1,2} \sum_{q=1,2} \alpha_{pq} \; I/K_{comp \,\in\, (I/K_p, MPK_q),t} + \eta_i + \epsilon_{i,t+5}, \tag{10}$$

where $\log \frac{Y_{i,t+5}}{Y_{i,t}}$ represents the 5-year log growth rate in sales, capital, gross profits, or total factor productivity (TFP)[17] of firm i from year t to $t{+}5$, $I/K_{i,t}$ denotes the investment rate of the firm i in year t, $I/K_{comp\,\in\,(I/K_p,MPK_q),t}$ refers to the median investment rate of competitor firms operating in the same industry as firm i and belonging to the portfolio $(I/K_p, MPK_q)$, and η_i represents a firm fixed effect, capturing unobserved heterogeneity specific to each firm. Hence, the regression model employs time-series variation to estimate the effect of a firm's own investment on its future growth (α_1) and the impact of its industry competitors' investments, depending on their position within the joint distribution of investment and marginal product of capital (MPK) (α_{pq}).[18]

Table 9 reveals significant positive effects of firms' own investment on sales growth, capital growth, profit growth, and a higher ranking in the future total factor productivity (TFP) distribution. Notably, the impact of competitors' investment yields striking results. While investments from other portfolios do not consistently predict firm growth, a higher median investment by competitor firms in the $(I/K_2, MPK_1)$ portfolio exhibits a significant negative relationship with growth and predicts a lower TFP ranking for the firm. These findings align with the research conducted by Kogan, Papanikolaou, Seru, and Stoffman (2017) and the empirical findings presented in Section 2, which indicate that firms in the $(I/K_2, MPK_1)$ portfolio tend to engage in a higher degree of innovative activities and make notable contributions, as evidenced by patent values and citations. Interestingly, the use of a broad measure of investment, rather than precise indicators of innovation, proves sufficient to capture the effect of competitors on firm growth. This further supports the notion that firms in the $(I/K_2, MPK_1)$ portfolio hold a distinct role in the economy compared to other firms. Their observed high investment rates are motivated by future prospects, encompassing both

[17] While sales, capital and profits reflect growth rates, TFP represents the level of TFP projected 5 years ahead, rather than its growth rate. The firm level TFP estimates used in this analysis are obtained from İmrohoroğlu and Tüzel (2014). These estimates are cross-sectional and should not be directly compared across different years.

[18] This regression specification is similar to the approach taken by Kogan, Papanikolaou, Seru, and Stoffman (2017), who examine the influence of firms' own and competitors' innovation activities on firm growth.

Table 9: Effect of Investment on Firm and Competitor Growth

This table presents the point estimates of Equation 10 for firm sales, total capital, gross profits, and TFP. The analysis relates firm growth and productivity both to the firm's own investment and the median investment rate of firms in the same SIC2 industry within each portfolio. The regressions include firm fixed effects and the corresponding t-statistics are reported in parentheses. The standard errors are clustered by firm and year, and corrected for serial correlation using the Newey-West correction with 8 lags. Statistical significance is denoted by one, two, or three stars, indicating significance at the 10%, 5%, and 1% levels, respectively. The variable N represents the count of firm-year observations, while R^2 indicates the adjusted R-squared value.

	(1) $\log \frac{Sale_{t+5}}{Sale_t}$	(2) $\log \frac{K_{t+5}}{K_t}$	(3) $\log \frac{Profit_{t+5}}{Profit_t}$	(4) $\log TFP_{t+5}$
I/K_{firm}	0.383***	0.590***	0.326***	0.060***
	(9.62)	(12.78)	(6.69)	(4.19)
$I/K_{comp \in (I/K_1, MPK_1)}$	−0.366	1.092	−0.078	0.015
	(−0.43)	(1.33)	(−0.08)	(0.08)
$I/K_{comp \in (I/K_1, MPK_2)}$	0.398	1.414***	0.143	0.347***
	(1.38)	(5.47)	(0.46)	(3.68)
$I/K_{comp \in (I/K_2, MPK_1)}$	−0.345**	−0.398***	−0.323**	−0.062***
	(−2.58)	(−3.37)	(−2.23)	(−3.49)
$I/K_{comp \in (I/K_2, MPK_2)}$	0.370	0.426*	0.330	0.128*
	(1.67)	(1.95)	(1.37)	(1.92)
Firm FE	x	x	x	x
R^2	0.015	0.085	0.009	0.004
N	127,168	127,168	119,629	92,942

their own growth prospects and the displacement of competitors.

The negative effect of $(I/K_2, MPK_1)$ portfolio investment on competitors can be attributed to the dominance of competition and creative destruction channels, which outweigh the potential benefits of technology spillovers that are not accounted for in our model. Consequently, the TFP gains derived from our model's perspective can be considered an upper bound in quantitative terms. However, a crucial question remains as to whether the negative effect on competitors is significant enough to entirely offset the advantages of increased investment stemming from the expectation of substantial improvements in a firm's own productivity.

To address this question, we analyze the relationship between firms' investment rates in portfolios sorted by I/K and MPK and future aggregate TFP growth. We utilize business sector and utilization-adjusted TFP measures developed by Fernald (2014) and estimate the following regression model:

$$\log \frac{TFP_{t+5}}{TFP_t} = \alpha_0 + \sum_{p=1,2} \sum_{q=1,2} \alpha_{pq} \ I/K_{(I/K_p, MPK_q)} + \sum_{l=0}^{2} c_l \ \log TFP_{t-l} + \epsilon_{i,t+5}. \tag{11}$$

Here, $\log \frac{TFP_{t+5}}{TFP_t}$ represents the 5-year log growth rate in aggregate TFP, and $I/K_{(I/K_p, MPK_q)}$ represents the median investment rate of firms belonging to the $(I/K_p, MPK_q)$ portfolio. Additionally, we incorporate controls for the three lagged values of $\log TFP$.[19]

Table 10 presents the results of univariate regressions, showing that median investment rates in low-investment portfolios $(I/K_1, MPK_1)$ and $(I/K_1, MPK_2)$ do not predict future TFP growth. However, the investment rates of above-median investment portfolios $(I/K_2, MPK_1)$ and $(I/K_2, MPK_2)$ positively predict future aggregate growth using both TFP measures. Given the common element in investment rates across high-investment portfolios, we perform a bivariate predictive regression using the median investment rates of $(I/K_2, MPK_1)$ and $(I/K_2, MPK_2)$. The investment rate of the $(I/K_2, MPK_1)$ portfolio dominates that of $(I/K_2, MPK_2)$ and remains a significant and positive predictor of aggregate TFP. This finding suggests that the high investments made by low MPK firms in the $(I/K_2, MPK_1)$ portfolio contribute to overall productivity growth in the economy, and the observed effect is economically significant: A one-standard deviation increase in the median investment rate of firms in the $(I/K_2, MPK_1)$ portfolio corresponds to an aditional

[19] A similar specification was used by Kogan, Papanikolaou, Seru, and Stoffman (2017) to examine the impact of economy-wide innovation measures on aggregate growth.

approximately one standard deviation increase in 5-year TFP growth.

The positive predictability of aggregate TFP by the investments of the $(I/K_2, MPK_1)$ portfolio indicates that the negative effect on competitors, as documented in Table 9, is outweighed by the positive effect on firms' own growth within the $(I/K_2, MPK_1)$ portfolio. In summary, while our previous sections focused on the impact of high investment-low MPK firms' investment on their own growth, the positive association between these firms' investments and aggregate TFP provides qualitative support for our counterfactual calculation in Figure 2, which suggests a reduction in aggregate TFP in the absence of "investing in misallocation."

5 Unpacking the $(I/K_2, MPK_1)$ Portfolio

In this section, we delve deeper into the nature of firms that invest heavily despite having a low MPK. The observation that these firms tend to be relatively young and focused on investing and growth aligns with the characteristics of growth firms. Empirical asset pricing studies have suggested that growth firms and firms with high investment and low profits, which are likely to overlap significantly with the high investment-low MPK portfolio, tend to have lower expected equity returns (Hou, Xue, and Zhang, 2015; Fama and French, 2015). However, we find that despite vastly different investment patterns, all four portfolios have very similar expected returns as shown in Table 2. This suggests that the high investment-low MPK portfolio used in our empirical analysis and estimation does not correspond to high investment-low profitability firms that typically have low expected returns.

We use linear factor models to explore the factors driving the average portfolio returns. In particular, factor models take the form $\mathbb{E}[R_t^e] = \alpha + \beta' X_t$ where β represents factor exposures and X_t includes factors. A model that prices the portfolio returns well (i.e., $\alpha = 0$) can provide insight into the sources of expected portfolio returns and the characteristics of the

Table 10: Effect of Investment on Aggregate Productivity

This table presents the point estimates of Equation 11, which examines the relationship between future aggregate productivity and the median investment rate in I/K and MPK-sorted portfolios. Panel A measures aggregate productivity using business sector TFP, while Panel B utilizes utilization-adjusted TFP. The regressions include controls for 3-lags of log TFP to account for the influence of past productivity levels. The standard errors are corrected for serial correlation using the Newey-West correction with 8 lags. The corresponding t-statistics are reported in parentheses, and statistical significance is indicated by one, two, or three stars, representing significance at the 10%, 5%, and 1% levels, respectively.

	(1)	(2)	(3)	(4)	(5)
			Panel A: Business sector TFP		
$I/K_{(I/K_1,MPK_1)}$	−0.027				
	(−0.15)				
$I/K_{(I/K_1,MPK_2)}$		0.097			
		(0.65)			
$I/K_{(I/K_2,MPK_1)}$			0.094***		0.118***
			(8.18)		(3.58)
$I/K_{(I/K_2,MPK_2)}$				0.097***	−0.031
				(5.07)	(−0.57)
			Panel B: Utilization-adjusted TFP		
$I/K_{(I/K_1,MPK_1)}$	0.168				
	(0.66)				
$I/K_{(I/K_1,MPK_2)}$		0.315			
		(1.51)			
$I/K_{(I/K_2,MPK_1)}$			0.125***		0.118***
			(7.52)		(3.48)
$I/K_{(I/K_2,MPK_2)}$				0.139***	0.009
				(6.78)	(0.18)

firms included in these portfolios.

Table 11 presents the results from two prominent factor models, the Fama-French 5-factor model Fama and French (2015) and the q^5 model of Hou, Mo, Xue, and Zhang (2021). Panel A shows that firms in the $(I/K_2, MPK_1)$ portfolio have substantial negative loadings on the HML and RMW factors, indicating that they tend to be growth firms with low profitability. However, two of the four portfolios, including the $(I/K_2, MPK_1)$ portfolio, have significant α estimates, indicating that the Fama and French (2015) model fails to capture the equal average returns of the I/K and MPK-sorted portfolios.

Panel B of Table 11 presents the factor loadings and alphas obtained from the q^5 model proposed by Hou, Mo, Xue, and Zhang (2021). This model is based on the investment CAPM framework and incorporates various factors aimed at capturing size, investment rate, and profitability. Additionally, it introduces an extra factor referred to as "expected growth," which predicts higher returns based on a dynamic q-theory model. Essentially, this implies that firms currently making higher investments exhibit lower returns, while firms expected to increase their investments in the future, all else equal, are associated with higher expected returns. Firstly, we note that all the alphas are small in magnitude and statistically insignificant, suggesting that the q^5 model effectively prices double-sorted portfolios on I/K and MPK. Furthermore, examining the factor loadings provides insights into the expected return sources for the high investment-low MPK portfolio. As hypothesized, the portfolio return displays negative loadings on both the investment and profitability factors.[20] However, this is counterbalanced by a positive and statistically significant loading on the expected growth factor, which exhibits the highest average returns among all the q^5 factors. This finding sheds light on the elevated returns of the high investment-low MPK portfolio. In our model, this portfolio consists of high-type firms that presently invest with the expectation of rapid

[20] The investment factor is long low and short high investment firms, the profitability factor is long high and short low profitability firms, and the expected growth factor is long high and short low expected growth firms.

future growth, aligning with these firms displaying a negative loading on the investment factor and a positive loading on the expected growth factor in the data.

While portfolios in the data have varying exposures to risk factors, we prioritize simplicity in developing our model by abstracting from aggregate risk. We incorporate empirical evidence of minimal variation in the average discount rates across portfolios suggesting that our portfolio level results are not driven by the relation between discount rates, investment, and MPK. Therefore, we assume a constant discount rate in our simple model that can capture the empirical relationship between firms' investment, MPK, and the growth of their cash flows in the absence of a discount rate channel.

6 Conclusion

This paper presents a novel empirical finding within Compustat firms: around 20% of these firms demonstrate above-median investment rates despite having below-median marginal product of capital. Within the conventional neoclassical framework, which views differences in MPK as indicators of misallocation to be avoided, these firms can be identified as misallocating resources. We argue that such "investing in misallocation" is not necessarily inefficient and is geared towards future growth. In particular, these firms possess distinct characteristics, such as being relatively young, and a small proportion (around 4%) experience significant jumps in their sales and MPK in the subsequent year, a rarity among other firms.

To capture the dynamics of heterogeneous firms, we propose and estimate a simple endogenous firm growth model. This model incorporates the idea that high-type firms' investments increase the likelihood of large upward moves in productivity. It aligns well with the observed data, explaining the high investment levels exhibited by firms despite their low MPK. Importantly, this model allows us to conduct counterfactual analyses by simulating a

Table 11: Portfolio Factor Regressions

This table displays the regression results of value-weighted excess returns for I/K and MPK-sorted portfolios on Fama-French and q^5-factor returns. Panel A presents the results for Fama-French factors, while Panel B presents the results for q^5 factors. The excess returns are measured on a monthly basis, covering the period from July 1976 to June 2017. The t-statistics for the coefficient estimates are presented in parentheses

	$(I/K_1, MPK_1)$	$(I/K_1, MPK_2)$	$(I/K_2, MPK_1)$	$(I/K_2, MPK_2)$
Panel A: Fama-French factors				
MKTRF	0.986***	0.996***	0.982***	1.055***
	(81.72)	(62.55)	(55.98)	(81.36)
SMB	0.034*	−0.009	0.043*	0.074***
	(1.90)	(−0.36)	(1.65)	(3.89)
HML	−0.070***	−0.071**	−0.320***	−0.205***
	(−3.06)	(−2.34)	(−9.67)	(−8.35)
RMW	0.173***	0.237***	−0.326***	−0.028
	(7.67)	(7.95)	(−9.93)	(−1.17)
CMA	0.292***	0.109**	0.068	−0.199***
	(8.30)	(2.35)	(1.33)	(−5.26)
α	−0.108**	−0.054	0.195***	0.094*
	(−2.18)	(−0.83)	(2.72)	(1.77)
Panel B: q^5 factors				
r_{Mkt}	0.963***	0.976***	1.014***	1.076***
	(73.04)	(59.91)	(52.33)	(75.11)
r_{Me}	−0.014	−0.070***	0.112***	0.064***
	(−0.75)	(−3.10)	(4.18)	(3.23)
$r_{I/A}$	0.176***	0.035	−0.416***	−0.412***
	(5.85)	(0.95)	(−9.41)	(−12.58)
r_{Roe}	0.014	0.186***	−0.317***	−0.036
	(0.57)	(6.20)	(−8.92)	(−1.36)
r_{Eg}	0.020	−0.103**	0.268***	0.051
	(0.54)	(−2.28)	(4.99)	(1.29)
α	−0.045	0.027	0.079	0.071
	(−0.77)	(0.38)	(0.91)	(1.11)

scenario in which firms are unable to invest in anticipation of such jumps. In this counterfactual scenario, we observe a better alignment between investment and MPK, resulting in a reduction in "misallocation." However, this adjustment also leads to a decrease in aggregate productivity.

Our contribution deviates from previous literature on misallocation in several aspects. While prior studies primarily focus on examining distortions and frictions such as adjustment costs, information asymmetries, and financial constraints that lead to misallocation, we take a distinct approach by considering the role of firms' investments in shaping their future productivity and show that some of the measured misallocation may be attributed to an overlooked aspect of optimal investment policy.

Furthermore, our analysis employs the Compustat sample, which consists of publicly traded firms with access to public capital markets. This distinguishes our study from earlier research that often concentrates on smaller private firms, frequently from emerging economies, (e.g., Midrigan and Xu, 2014; Bau and Matray, 2023). By examining publicly traded firms, which tend to be larger in scale, we mitigate the potential impact of financial constraints that can be more pronounced among smaller private firms. In fact, Farre-Mensa and Ljungqvist (2016) have documented that publicly traded firms typically face minimal financial constraints.

In summary, our paper offers a fresh perspective on the interpretation of MPK dispersion as an indicator of misallocation. Through our empirical analysis and the utilization of the Compustat sample, we provide new insights into the investment behavior and productivity dynamics of firms, contributing to a deeper understanding of resource allocation in the economy.

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Appendix

A Data and measurement

A.1 Sample and variable construction

Our sample includes all Compustat firms except regulated utilities (SIC codes between 4900 and 4999), financial firms (SIC codes between 6000 and 6999), and firms categorized as public service, international affairs, or nonoperating establishments (SIC codes 9000+). We also exclude firms with missing or non-positive book value of assets (AT), sales (SALE) and physical capital (PPEGT). As in Peters and Taylor (2017), our sample starts in 1975, because this is the first year that the Federal Accounting Standards Board (FASB) requires firms to report R&D. We winsorize all regression variables at the 0.5% level to reduce the impact of outliers.

We measure physical investment as the difference between capital expenditures and property sales, CAPX-SPPE; and physical capital from plant, property, and equipment (PPEGT). For measurements of intangible investment and capital we adopt Peters and Taylor (2017)'s definitions. Intangible investment is measured from R&D (XRD) and SG&A expenses (XSGA-XRD) as R&D + (0.3 × SG&A).[21] This definition assumes 30% of SG&A represents investment in intangible capital. Intangible capital is calculated by applying the perpetual inventory method to intangible investment and obtained from Peters and Taylor (2017). We define a firm's total investment and capital as the sum of its physical and intangible components, as in Peters and Taylor (2017). When data for CAPX, SPPE, XRD, or XSGA are missing, they are imputed as zero. The investment-to-capital ratios are computed by dividing the total investment in year t by the capital in year t-1.

[21] Compustat data item XSGA includes R&D expense reported in XRD. In order to isolate SG&A we subtract XRD from XSGA as in Peters and Taylor (2017).

Following David, Schmid, and Zeke (2022), we measure firm's marginal product of capital, MPK, in logs (up to an additive constant) as the difference between log sales and capital. All empirical results are based on a sample of firms for which we can compute both the MPK and investment-to-capital ratio (I/K).

We define firm jumps as situations where a firm experiences a doubling of sales, accompanied by a larger than 50% increase (40 log points) in its marginal product of capital. To reduce the impact of noise, we measure jumps over a two-year period. Specifically, we calculate current sales and MPK by averaging sales and MPK in years *t-1* and *t*, and future sales and MPK by taking the averages of years *t+1* and *t+2*. Instances where growth from current to future values exceeds our threshold values are considered as jumps.

We made several adjustments to our definition of jumps in order to minimize errors caused by M&A activity and double counting. First, we exclude jump instances that coincide with a significant merger activity (Compustat sales footnote SALE_FN=AB) in the years *t*, *t+1*, or *t+2*. Additionally, to account for the impact of smaller mergers on sales growth, we adjust sales for AQS (Acquisitions/Sales Contribution) by adding $AQS_t/2 + AQS_{t+1} + AQS_{t+2}$ to current sales, and $AQS_{t+2}/2$ to future sales before calculating the growth rate. Finally, we eliminate cases where two consecutive years of jumps overlap due to the use of two-year averages. While the probability of jumps may differ depending on the criteria applied, we have conducted tests to confirm the robustness of our results to modifying the jump cutoffs for sales and MPK growth. Appendix Table A.2 reproduces the findings from Table 1 using lower and higher jump thresholds. Furthermore, we have validated that the corrections made to address mergers and double counting do not substantially affect the outcomes.

Firm age is defined as the number of years since the firm was included in the Compustat database, FYEAR-YEAR1. Firms that have been in operation for less than 10 years are classified as "young firms," while companies that have been in operation for 10 years or more are classified as “mature firms,” following the classification of Haltiwanger, Jarmin, and Miranda (2013). This classification results in approximately half of the firm-year observations being categorized as young firms.

Table 2 presents various statistics measuring innovative activity, productivity, and financial constraints. The information on patent counts, economic values (derived from the stock market’s response to patent grants), and citations is sourced from Kogan, Papanikolaou, Seru, and Stoffman (2017). To measure breakthrough innovation, the counts for the top 20%, 10%, and 5% patents, which evaluate the textual similarity between a given patent and previous or subsequent patents, are obtained from Kelly, Papanikolaou, Seru, and Taddy (2021). Assessments of firm exposure to the Life1 stage (pertaining to product innovation) are based on textual analysis of 10-K files and are acquired from Hoberg and Maksimovic (2022). All patent-related measures are assigned to firms based on the year of patent filing. Firm-level TFP estimates are sourced from İmrohoroğlu and Tüzel (2014). Financial constraints are quantified using the SA index, which is computed by considering firm size and age coefficients as provided by Hadlock and Pierce (2010).

A.2 Target moments in the data

To quantify the cross-sectional dispersion in investment and output (sales), we employ the interquartile range (IQR). We compute the IQR for each year and then average the results for the period spanning 1975 to 2015. Details on the classification of young and mature firms and the definition of I/K (investment-to-capital ratio) can be found in Section A.1. Prior to calculating the IQR, the firm values are normalized to account for industry differences by subtracting the 2-digit SIC industry median values of I/K and sales growth from the corresponding firm values. The nonnegative investment share is the percentage of observations in which I/K is nonnegative.

To calculate the moments associated with jump realizations, we limit our sample to observations that exhibit jumps (as specified in Section A.1). We then determine the median sales jump ($\frac{Sale_{next}}{Sale_{current}}$) and log MPK jump ($log\frac{MPK_{next}}{MPK_{current}}$), measured from the *current* period (average of years *t-1* and t) to the *next* period (average of years *t+1* and *t+2*). Jump age is defined as the firm's age in year t.

To compute the moments associated with the $(I/K_2, MPK_1)$ portfolio, we employ a simultaneous sorting of Compustat firms based on I/K and MPK within each SIC2 industry and year. We restrict our attention to the observations that are placed in the $(I/K_2, MPK_1)$ portfolio to calculate the portfolio share, which represents the percentage of all observations in the portfolio, averaged over the years. Similarly, we calculate the portfolio share among young firms by restricting our analysis to firms identified as young. The jump probability is the time series average percentage of $(I/K_2, MPK_1)$ firms that experience a jump in a given year. The industry-adjusted median I/K of $(I/K_2, MPK_1)$ portfolio is computed by calculating the median industry-adjusted investment rate every year and averaging it over the years.

A.3 Exits in the data

We identify and categorize different types of firm exits (merger versus firm death) using Compustat exit codes. Specifically, we rely on the DLRSN ("Research Company Reason for Deletion") variable for classification purposes. Our approach assigns a merger classification to firms with exit codes 01 (Acquisition or merger), 04 (Reserve acquisition), 06 (Leveraged buyout), and 09 (Now a private company). Conversely, we classify firms with exit codes 02 (Bankruptcy), 03 (Liquidation), 05 (No longer fits the original format), 07 (Other), and 09 (Other) as having experienced a firm death event.

To identify a firm exit event, we look at the fiscal year of the last available data in

Compustat (i.e., fyear=year2). Based on these criteria, we find that the annual merger rate is 4%, while the annual firm death rate is 2.9% during our sample period.

Upon further analysis of the investment and MPK sorted portfolios, we observe that the merger rate is fairly consistent across the different portfolios, ranging from 3.8% for the (I/K_2, MPK_1) portfolio to 4.1% for the (I/K_1, MPK_2) portfolio. In contrast, the variation in firm death rates was more substantial, ranging from 2.1% for firms in the (I/K_2, MPK_2) portfolio to 3.8% for firms in the (I/K_1, MPK_1) portfolio. For firms in the (I/K_1, MPK_2) and (I/K_2, MPK_1) portfolios, the probability of a firm exiting via firm death is 2.6% and 2.9%, respectively. Based on this analysis, we assume a death probability of 2.9% in the model. It is worth noting that a higher death probability would have effects similar to increased capital depreciation in the model solution.

Mergers, on the other hand, do not impact the firm's policy function, as investors receive payment equal to the firm's value at the time of the merger. However, mergers do alter the composition of firms in the economy. Consequently, after solving the model, we incorporate an additional 4% to the death probability in simulations to account for mergers. This adjustment ensures that the simulated data reflects the characteristics observed in Compustat data.

B Model solution and estimation

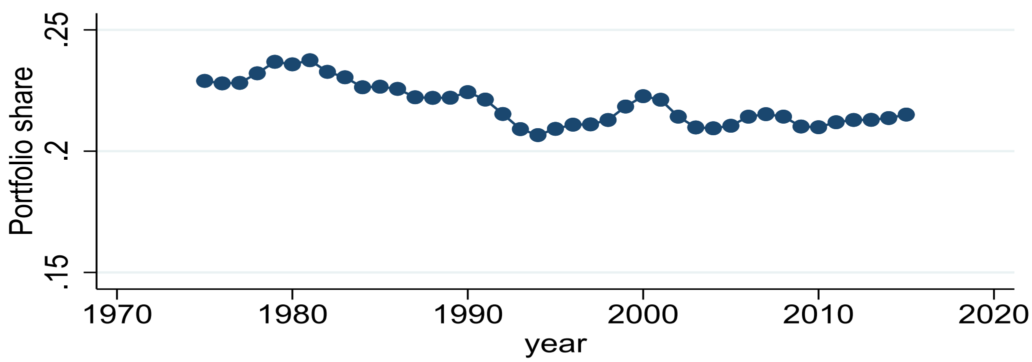
For the estimation process, we generate simulated data using a sample of 2,000 firms over a span of 2,000 years. To initialize the value of $k = K/Z$, we start with the steady-state value of k and introduce an initial shock (Gaussian and jump) to Z. Simulated annealing is employed to minimize the criterion function described in equation (9). To compute standard errors for the parameters, we utilize numerical derivatives of the simulation moments with respect to each parameter, weighted by the identity matrix.

The numerical solution for both types of firms' maximization problems is obtained using value function iteration. In this process, we define a grid of points for the state variable k as $k(i) = k(i-1) + c_1 \exp(c_2(i-2))$, where c_1 and c_2 are chosen to ensure that we have 200 grid points between the specified minimum and maximum values of k. Additionally, we discretize the Gaussian shock using ten grid points. Once the grids are established, we compute conditional expectations through interpolation and iteratively execute the value function maximization until there is no further change in the investment policy and value function.

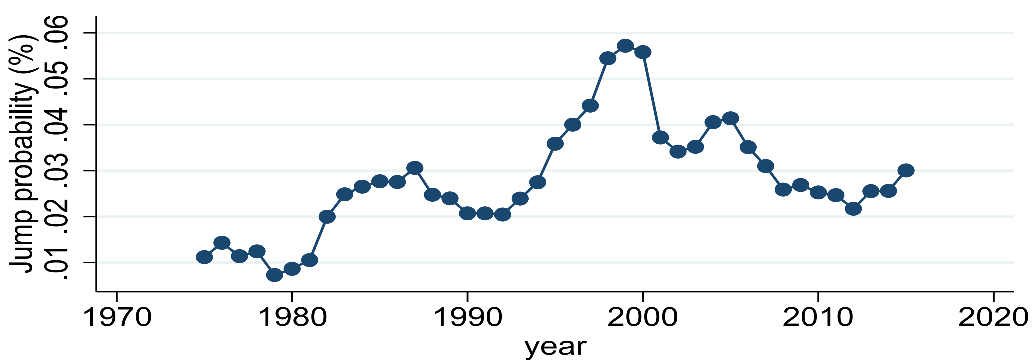
C Appendix Figures and Tables

Figure A.1. ($I/K_2, MPK_1$) Portfolio share and jump probability over 1975-2015 (Physical investment and capital only)

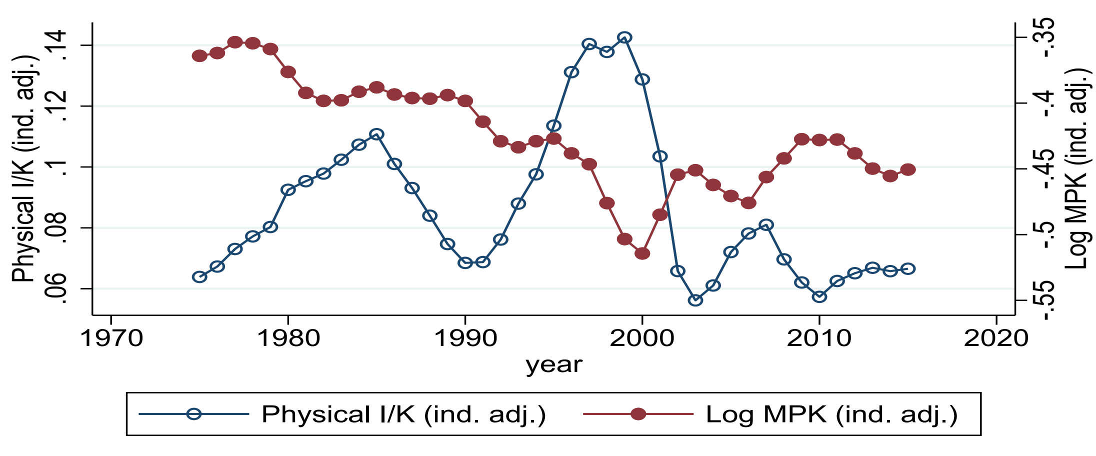
The figure illustrates the portfolio share, jump probability, and median investment rate and MPK for firms in the ($I/K_2, MPK_1$) portfolio between 1975 and 2015. Investment rate, MPK, and jump measures are based solely on physical investment and capital. Panel A presents the portfolio shares, Panel B displays the jump probabilities, and Panel C shows the industry-adjusted median investment rate and log MPK. All variables are presented as 3-year moving averages.



(a) Share of Compustat firms placed in ($I/K_2, MPK_1$) portfolio



(b) Annual jump probability of firms in ($I/K_2, MPK_1$) portfolio



(c) Median I/K and MPK of firms in ($I/K_2, MPK_1$) portfolio

Table A.1: Physical I/K and MPK Sorted Portfolios

To construct the four portfolios, all firms are annually sorted into below and above median physical I/K and MPK groups. The resulting portfolio statistics are presented in columns 1 to 4, while columns 5 and 6 display the differences between firms in the $(I/K_2, MPK_1)$ group and the entire sample and their $t-$statistics. For each variable, the statistics are initially computed for all firms within each portfolio and then averaged across years. Appendix A provides a detailed explanation of variable definitions and sources. To account for industry differences, physical I/K, log MPK, and log TFP are normalized by subtracting the median values of their respective 2-digit SIC industries, and the median SA index is normalized to be 1 each year to increase readability. Excess future stock returns $(r - r^f)$ are measured from July of year $t + 1$ to June of year $t + 2$. Most variables are reported as portfolio medians, except for TFP where the 90th percentile is also presented. Excess returns $(r - r^f)$ are calculated as value-weighted averages, while patent-based variables are reported as means due to the highly skewed nature of patenting activity.

	$(I/K_1, MPK_1)$	$(I/K_1, MPK_2)$	$(I/K_2, MPK_1)$	$(I/K_2, MPK_2)$	$(I/K_2, MPK_1)$–*All*	
					Difference	t–stat
Panel A: Portfolio properties						
N	1476.3	1110.8	1125.5	1429.8		
Physical I/K (median, ind. adj.)	-0.053	-0.051	0.086	0.10		
Log MPK (median, ind. adj.)	-0.50	0.48	-0.42	0.57		
Portfolio share	0.29	0.22	0.22	0.28		
Portfolio share among young firms (≤ 10 years)	0.23	0.20	0.24	0.33		
Age (median)	13.3	11.9	10.1	9.26	-0.91	-1.50
Jump probability (%)	0.015	0.0087	0.028	0.012	0.012***	5.21
Panel B: Innovative activity and product development						
Patents/K (mean)	10.9	10.4	20.6	21.0	4.83**	2.38
Patent Value/K (mean)	26.2	21.8	86.1	91.3	28.9**	2.07
Patent Citations/K (mean)	317.7	282.7	830.3	708.4	295.1**	2.42
Top 20% Patents/K (mean)	2.19	1.90	5.60	4.75	1.97**	2.42
Top 10% Patents/K (mean)	1.29	1.07	3.32	2.61	1.24**	2.24
Top 5% Patents/K (mean)	0.79	0.56	2.04	1.44	0.83**	2.19
Exposure to Life1 Stage (median)	0.21	0.21	0.25	0.25	0.017***	2.82
Panel C: Productivity, returns, financial constraints						
Log TFP (median)	-0.43	-0.38	-0.26	-0.19	0.052***	9.66
Log TFP (90th pctile)	-0.012	0.021	0.17	0.30	0.028	1.32
Log Future TFP (5yr later, median)	-0.38	-0.35	-0.29	-0.27	0.034***	4.70
Log Future TFP (5yr later, 90th pctile)	0.070	0.078	0.20	0.24	0.044*	1.91
Excess future stock returns (VW mean, annual, %)	8.53	8.76	7.61	7.92	-0.27	-0.068
SA index (median)	-0.097	0.11	-0.11	0.087	-0.11***	-5.62

Table A.2: Determinants of Firm Jumps with Alternative Jump Thresholds

This table presents coefficient estimates obtained from linear probability models analyzing realized jumps with alternative jump definitions. The dependent variable is the jump dummy, which takes a value of 1 when a jump occurs from the current period to the next period. Panel A presents results with a lower jump threshold, where jumps are defined as cases where sales increase by at least 50% and MPK rises by 30 log points. Panel B presents results with a higher jump threshold, where jumps are defined as cases where sales increase by at least 150% and MPK rises by 50 log points. For a comprehensive explanation and the definition of explanatory variables, please refer to Appendix A. The regressions incorporate 2-digit SIC industry-year fixed effects. The corresponding t-statistics are presented in parentheses, and standard errors are clustered at the firm-year level. Statistical significance levels are indicated by one, two, and three stars, denoting significance at the 10%, 5%, and 1% levels, respectively. The variable N denotes the count of firm-year observations, while R^2 represents the adjusted R-squared value.

		Panel A: Lower jump threshold			
	(1)	(2)	(3)	(4)	(5)
Physical I/K		0.023***	0.022***		
		(11.21)	(12.28)		
Intangible I/K		0.018***	0.029***		
		(5.63)	(10.11)		
Total I/K				0.048***	0.032***
				(15.95)	(9.55)
Log MPK	−0.032***		−0.034***	−0.034***	−0.034***
	(−20.58)		(−23.07)	(−22.64)	(−23.10)
Log age					−0.015***
					(−20.55)
Ind × Year FE	x	x	x	x	x
R^2	0.045	0.032	0.052	0.051	0.055
N	197,749	197,510	197,510	197,749	197,749
		Panel B: Higher jump threshold			
	(1)	(2)	(3)	(4)	(5)
Physical I/K		0.016***	0.015***		
		(10.21)	(11.49)		
Intangible I/K		0.013***	0.019***		
		(5.84)	(8.50)		
Total I/K				0.036***	0.031***
				(12.91)	(11.22)
Log MPK	−0.016***		−0.017***	−0.017***	−0.017***
	(−12.73)		(−14.46)	(−14.58)	(−14.71)
Log age					−0.005***
					(−11.02)
Ind × Year FE	x	x	x	x	x
R^2	0.034	0.026	0.046	0.046	0.047
N	197,711	197,472	197,472	197,711	197,711