Bank Runs and Privately Funded Solutions

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Abstract

In a simple extension of the Diamond and Dybvig (1983) environment, we compare three regimes: an explicitly modeled privately funded deposit insurance scheme, an ex-ante funded liquidity insurance scheme, and runs preventing bank contracts without any insurance. It is shown that when the probability of runs is low, both insurance schemes are superior to runs preventing contracts; and in this case, the deposit insurance scheme is socially desirable as long as asset liquidation costs are not excessively high. When liquidation costs are high, the liquidity insurance scheme outperforms the other two regimes. And when the probability of runs is high, the arrangement with runs preventing bank contracts is the best among the three regimes. We also show that when depositors are sufficiently risk averse, the deposit insurance scheme is socially the most desirable among the three regimes.

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1 Introduction

Following the collapse of Silicon Valley Bank and Signature Bank in March of 2023, the heads of the U.S. Treasury, Federal Reserve and Federal Deposit Insurance Corporation (FDIC) stated in a joint press release that in the rescue of these financial institutions "no losses will be borne by the taxpayer" (Federal Reserve Board, March 12, 2023). Separately, President Joe Biden assured the public: "Let me repeat that: No losses will be borne by the taxpayers" (New York Times, March 13, 2023). These steps seem to be taken to prevent a reaction similar to the public outcry following the bank bailout of 2008 (e.g., Los Angeles Times, September 26, 2008). Given the apparent sensitivity of both society and government to the issue of using the public purse to rescue financial institutions, it would be of interest to explore privately funded solutions to the problem. In this paper, we analyze three distinct privately funded approaches to mitigating the phenomenon of bank runs–deposit insurance (DI) scheme, liquidity insurance (LI) scheme and run preventing contracts (RPC)–and examine the specific conditions under which each of these solutions proves most effective. These approaches align with three historical banking frameworks, each representing a different strategy for managing financial stability and preventing bank runs.

The root of any bank run is intimately connected to the economic function of banks. After inspecting the asset and liability sides of bank balance sheets one quickly comes to the following observation: bank assets are illiquid while liabilities are extremely liquid – demand deposits allow depositors to withdraw funds at any time. This liquidity transformation is viewed as one of the key missions of modern banks. But it is not without risks. On the one hand, it allows banks to pool resources from retail depositors and finance long-term projects with high returns. On the other, the liquidity mismatch is a root of banks' problems: they are vulnerable to the possibility of a bank run. During such an event, many depositors rush to the bank to withdraw deposits at once, and the suddenly large number of withdrawals force banks to liquidate their long-term assets at fire sale prices. When the liquidation value of assets is not enough to cover the value of deposits demanded by bank customers, banks have to go bankrupt and some depositors may lose their deposits. Bank runs can be particularly harmful when banks in trouble do not have a solvency problem. In such a case a bank run is entirely expectations-driven and a healthy institution is unnecessarily destroyed: depositors with no need for cash at the time suddenly rush to withdraw only because they believe other depositors will do the same.

Bank runs can cause severe financial damage to the affected depositors and banks. Moreover, bank runs can destabilize the entire banking system since runs can be contagious. As argued in the seminal work by Diamond and Dybvig (1983) – DD hereafter – "in a panic with many bank failures, there is a disruption of the monetary system and a reduction in production." And DD propose the use of deposit insurance to address the problem of bank runs. It has been widely recognized for some time that deposit insurance (DI) plays a crucial role in maintaining depositor confidence, and as a result, it has become a widely adopted solution. For example, in 2013, around

85% of high-income countries and 60% of upper-middle-income ones had explicit DI, with the total number of countries with this arrangement reaching 112 (see Demirgüç-Kunt et al. 2015).

The type of DI that DD propose and analyse is government deposit insurance funded through taxation. In the case of a run, the government can tax early withdrawers and plow the collected resources back to the bank in such a manner that no depositor has an incentive to run. Thus, the government-funded deposit insurance scheme eliminates bank runs as a possible equilibrium. Despite the clear advantage of government-funded deposit insurance in terms of credibility as it is backed by the government's ability to tax, in most countries deposit insurance is privately funded by participating banks. As described in Demirgüç-Kunt et al. (2015), which provides a comprehensive global database of DI arrangements as of 2013, around 77% of all DI schemes around the world and 91% of DI schemes in high-income countries are privately funded. Furthermore, most DI schemes are ex-ante funded (as opposed to the ex-post funded DI proposed in DD), i.e. they have a fund built by insurance premiums paid by banks on a scheduled basis. For example, according to IADI (2018), between 2008 and 2014, the percentage of DI schemes with ex-ante funding increased from 83% to 90%. Although some privately funded DI programs have either implicit or explicit government support in the form of a backstop, it is usually not in the form of taxation but rather takes a form of lines of credit or issuance of bonds or loans guaranteed by the government. In addition, many OECD countries do not have such backstops.[1]

It is important to explicitly model ex-ante privately funded DI programs given how prevalent they are around the world. Despite the vast literature on bank runs and related remedies, only few papers model privately funded deposit insurance. Dávila and Goldstein (2023) model deposit insurance as a scheme financed by government through taxation as in DD; and they study an extension of the model where the deposit insurance fund is financed by bank contributions, as in our model. However, they treat bank contracts as a primitive of the model to simplify the analysis. Oosthuizen and Zalla (2022) extend the model of Dávila and Goldstein (2023) to embed it in an infinite-horizon framework. In the current paper, we modify the environment in Cooper and Ross (1998), which is an extension of DD that accounts for the positive costs associated with liquidating assets. Unlike Dávila and Goldstein (2023) and Oosthuizen and Zalla (2022), we do not have taxpayers in the model and assume that the DI fund is sufficient to cover all the losses involved. And the DI fund is built ex ante by contributions from participating banks. The combined contributions from banks are only sufficient to reimburse depositors of distressed financial institutions who cannot recover their deposits due to their bank's failure. The DI is incomplete in that all depositors receive the amount promised to the early type.[2] As in DD, a bank run is an equilibrium in the baseline model without DI. This implies that once the private DI scheme is

[1]This includes Canada, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Spain, Sweden, Switzerland. See Demirgüç-Kunt et al. (2015).

[2]Most DI schemes exhibit some sort of incompleteness. For example, FDIC in the U.S. imposes a $250,000 cap on deposit insurance.

introduced, depositors are protected but bank runs are still possible because of incompleteness of the DI. We assume that a portion of banks will encounter a run, requiring them to sell off their assets to pay depositors. As in Cooper and Ross (1998), liquidation is costly; when a troubled bank runs out of resources it declares bankruptcy, and the DI scheme pays the remaining bank customers. The feature of our model that allows bank runs and bank failures in the presence of a DI scheme is consistent with occasional bank runs in countries with DI. For instance, between March and May of 2023, there were bank runs at Silicon Valley Bank, Signature Bank and First Republic Bank, all U.S. institutions, as well as Credit Suisse, a bank located in Switzerland (New York Times, May 1, 2023 and April 24, 2023). Earlier examples in the U.S. include the failure of Continental Illinois in 1984, a bank holiday in the state of Ohio in 1985 in the wake of a run on Home State Savings Bank, and the collapse of Washington Mutual in 2008; a run on Northern Rock in 2007 in the U.K. was a harbinger of the upcoming global financial crisis (see FDIC 1997; Bosworth et al. 1985; New York Times, May 1, 2023; and Shin 2009 respectively).

In a bank run, DI protects depositors but not banks: the DI authority steps in only after a bank failure. To further understand the circumstances under which the DI scheme is socially desirable, we introduce another privately funded insurance scheme that we dub 'liquidity insurance'. As in the case of DI, liquidity insurance (LI) is ex-ante funded by contributions from participating banks. In contrast to deposit insurance, LI safeguards financial institutions by providing them with liquidity support, backed by collateral, during times of distress. The LI payout is sufficient to deter bank runs because banks do not have to liquidate assets at all, which keeps those who plan to withdraw late from running on their bank. As a result, by shielding banks, the LI scheme also protects depositors, making DI unnecessary.[3] The role of a lender of last resort (LLR) is sometimes referred to as "liquidity insurance". In fact, historically, institutions other than central banks played the role of LLR. For example, the Suffolk Bank played such a role during the Panic of 1837 and its aftermath. Thus, our second regime may be viewed as a privately arranged alternative to the LLR function of central banks.[4,5] The LI regime in our paper is also somewhat related to mutual liquidity risk insurance in Calomiris et al. (2015), in which there exists interbank liquidity assistance among banks but banks impose on each other minimum cash requirements enforced by a clearing house.[6] However, in our model, all extra liquidity is held at a central location (the liquidity insurance authority).

One useful framework for conceptualizing the LI regime is to compare it to historical clearinghouses. In the United States, between 1853 and 1913, clearinghouses were private associations

[3]Banks usually fail due to reasons other than bank runs. And the role of DI in such cases is essential. In our simple model, DI exists only to protect depositors in the case of a run, which, in turn, is the only reason banks fail. Thus in our context, LI replaces the role of DI only in this capacity.

[4]Central banks, the modern day LLRs, usually lend against collateral. So does the LI authority in this paper.

[5]Repullo (2000) and Kahn and Santos (2005) also consider the possibility of the role of LLR being played by the DI provider.

[6]See Sleet and Smith (2000) on the interplay between LLR and DI and Santos (2006) for a review of literature on this issue.

composed of member banks, providing critical emergency liquidity to their members during financial panics, backed by collateral (Jaremski, 2015; Kroszner, 2000). These institutions played a pivotal role in stabilizing the financial system during periods of distress. The LI scheme proposed in our analysis can be viewed as an evolved or 'strengthened' version of these historical clearinghouses. Unlike traditional clearinghouses, which primarily offered liquidity support, the LI regime is designed to effectively prevent bank runs altogether. This preventative capability significantly extends beyond the historical role of clearinghouses, which managed liquidity crises but did not necessarily eliminate the risk of bank runs. By ensuring that banks have access to necessary liquidity before a crisis escalates into a run, the LI regime provides a more robust safety net for the banking sector.

The final regime that we analyze as a solution to bank runs does not involve any form of insurance. Each bank offers a runs preventing contract (RPC), according to which the bank puts aside enough liquidity to be able to stave off a run; it is a form of self-insurance. The RPC scheme is somewhat related to the literature on narrow banking, which supports the 100% reserve requirement for banks (see Cochrane 2014 and Rodriguez Mendizábal 2020). This solution to financial fragility has been criticized on many grounds (see, e.g., Diamond and Dybvig 1986 and Wallace 1988). For example, Wallace (1988) views the idea of narrow banking as "synonymous with preventing banks from carrying out their main function." Unlike the 100% reserve requirement, the RPC promises the early withdrawers a relatively low amount that allows a bank to both keep a sufficient amount of liquidity (but not 100% of deposits) to survive an event in which all depositors decide to withdraw early, and to make loans (in the model, to invest in a productive technology). Although the RPC scheme completely eliminates bank runs, there is an opportunity cost of 'idle' liquidity. In addition, banks (collectively) do not take advantage of aggregate information on the probability of runs.

In summary, the three banking regimes analyzed in our study can be conceptualized as follows:

- RPC regime: This system exemplifies a laissez-faire banking model, reminiscent of the U.S. free banking era. Rolnick and Weber (1984) provide a detailed historical account of this era, highlighting its minimal regulatory intervention and the high degree of autonomy afforded to banks. In this regime, we assume that banks adopt a highly conservative investment strategy, aiming to prevent and withstand any potential bank runs.

- LI regime: This regime is analogous to the function of clearinghouses in the U.S., as described earlier. These institutions historically provided emergency liquidity to member banks during financial panics, backed by collateral. The LI regime in our analysis represents an enhanced version of these clearinghouses, designed to more effectively prevent bank runs through proactive liquidity support.

- DI regime: This regime mirrors the modern framework of deposit insurance schemes, exemplified by the Federal Deposit Insurance Corporation (FDIC) in the United States. For a comprehensive overview of the impact and mechanics of such schemes, see Demirgüç-Kunt et al. (2015). These schemes are designed to protect depositors by insuring their deposits against bank failures, thereby promoting financial stability and reducing the likelihood of bank runs.

Each of these regimes offers a distinct approach to banking regulation and support, reflecting different historical contexts and policy objectives.

In our analysis of these three regimes, we first establish that when the probability of bank runs α is large, it is best for a bank to self insure using the RPC; and conversely, when α is small, it is socially desirable to have either DI or LI. The intuition behind this result is as follows. When α is large, the cost of funding an insurance scheme (DI or LI) is large and outweighs the cost of self insurance under the RPC, which does not depend on α. Similar reasoning explains the advantages of the insurance schemes when α is small: banks put aside too much liquidity under the RPC, which is not socially desirable. Putting it differently, banks in the RPC regime do not take advantage of the low probability of runs, which is better handled by an insurance scheme.

Next, for relatively small values of α, we demonstrate that LI outperforms DI when the cost of liquidation τ is high. A key observation here is that the allocation of resources under LI does not depend on τ: no liquidation happens under this regime. When τ increases, the burden of liquidation in the DI scheme becomes too high, which makes LI a superior choice.

One notable finding of our study indicates that the DI scheme is socially preferable over the other two schemes when both the probability of bank runs α and the liquidation costs τ are low. The superiority of DI over the RPC follows from the first result above. When comparing DI with LI, we have to take into account two forces. On the one hand, there are liquidation costs that some banks have to bear in the DI case since the probability of having a bank run is positive (and it is zero in the case of the LI scheme since bank runs are eliminated); on the other, banks pay lower insurance premiums in the DI scheme than they do under the LI scheme. When liquidation costs are low, the latter effect outweighs the former leading us to the conclusion that the DI scheme is socially preferable to LI. In addition, our numerical analysis extends this result further by demonstrating superiority of DI for modest values of α and low-to-medium levels of τ.

The remaining of the paper is organized as follows. Section 2 presents the model and main arrangements we consider in the paper. Section 3 presents the main results for a general utility function. Section 4 considers the CRRA utility function and presents new insights. Section 5 presents two extensions of the main model. Section 6 summarizes and concludes. Proofs are in the Appendix.

2 Model

We study an environment closely related to that introduced by Cooper and Ross (1998), which is a version of the model pioneered by DD. Consider an economy with a continuum of agents of measure one who live for three periods, $t = 0, 1, 2$. There is also a continuum of banks of measure one. An agent can prefer to consume in either period 1 (they are called the early type) or period 2 (the late type). Agents' types are not known in period 0 and only revealed in period 1. However, it is known that a fraction π of agents will be the early type and the complementary fraction $1 - \pi$ the late type. Each agent is endowed with one unit of goods in period $t = 0$. There are two technologies in the economy. The first, called storage, takes one unit of goods in period t and returns one unit in period $t + 1$, $t = 0, 1$. The second, called the productive technology, takes one unit in $t = 0$ and returns R units in $t = 2$, where $R > 1$. If the process in this technology is disrupted in $t = 1$, one obtains only $1 - \tau$ units for each unit invested in $t = 0$, where $0 \leq \tau \leq 1$. Here τ is interpreted as the liquidation cost of long-term assets. In DD this cost was assumed to be zero.

In period $t = 0$, each agent deposits their endowment into a bank. The utility of consumption is $u(c_i)$, where c_1 and c_2 denote consumption in $t = 1$ by the early type and in $t = 2$ by the late type respectively. We assume that function $u(\cdot)$ is increasing and strictly concave with $u'(0) = \infty$ and $u(0) = -\infty$.[7]

We will make the following assumption for the rest of the paper.

Assumption 1. The coefficient of relative risk aversion is greater than unity everywhere: $-u''(c) \cdot c/u'(c) > 1$ for any $c \geq 0$.

Assumption 1 is needed to make sure that the first-best allocation, the solution to problem (1)–(2) below, entails existence of a bank run equilibrium. Intuitively, more risk averse agents want higher levels of consumption smoothing across two possible type realizations, and under Assumption 1, the consumption level c_1 becomes so close to c_2 that it exceeds unity: $c_1 > 1$; and this implies that a bank will have insufficient resources to satisfy depositors' demand if everyone panics in period $t = 1$.[8] This, in turn, justifies the need for deposit insurance, as in Diamond and Dybvig (1983).

Throughout the paper we assume, similar to Cooper and Ross (1998), that with probability α a bank run occurs at a given bank; since there is a mass one of banks, the fraction of banks that experience a bank run is also α.

2.1 The first-best allocation

Facing uncertainty regarding their type, each agent who chooses to invest on their own, maximizes an expected utility subject to technology constraints. However, DD have shown that a better

[7]For example, the CRRA function $u(c) = c^{1-\eta}/(1-\eta), \eta > 1$ satisfies these conditions.

[8]Diamond and Dybvig (1983) use this assumption to show existence of a bank-run equilibrium in the case $\tau = 0$. We keep the assumption as it helps in the case $\tau > 0$ too. See Ennis and Keister (2009) for a useful discussion on this issue.

arrangement, in fact the social optimum, can be attained by pooling resources and offering insurance against the uncertainty of type realization. The related social planner's problem is

$$\max\,[\pi\, u(c_1) + (1-\pi)\, u(c_2)], \tag{1}$$
$$\text{s.t.}\quad \pi\, c_1 = 1 - i \quad \text{and} \quad (1-\pi)\, c_2 = iR\,.$$

The two constraints above can be combined to yield

$$\pi c_1 + \frac{1-\pi}{R}\, c_2 = 1\,. \tag{2}$$

The first-order condition is $u'(c_1) = Ru'(c_2)$. It is easy to show that the solution satisfies $c_1 < c_2$. Let us denote this first-best allocation as (c_1^f, c_2^f).

The first-best allocation can be decentralized by a bank contract whereby agents deposit their endowments to a bank and the bank promises c_1^f to those who claim early consumption and c_2^f to those who claim late consumption. DD have shown that $1 < c_1^f < c_2^f < R$ if the coefficient of relative risk aversion is greater than unity everywhere (Assumption 1 above). The implication of this is that in this environment there exists an equilibrium called a 'bank run' in which all late types claim to be early types. In such a case, a bank will run out resources to satisfy depositors' demand because the total amount of resources is $1 - \tau i$, which is less than the total amount they have to pay out: $c_1 > 1$. To address this issue, Diamond and Dybvig propose a government deposit insurance funded by taxation.

2.2 Privately funded deposit insurance

We consider arrangements alternative to government-funded deposit insurance. As pointed out in the introduction, most DI schemes are ex-ante privately funded programs. For example, in the EU, none of the DI schemes are government funded and only few are funded jointly between the government and private actors (IMF 2013). Two alternative insurance schemes funded privately by bank ex-ante contributions are studied in this paper. The idea behind the first one, deposit insurance (DI), is that the total bank contributions to the DI fund should be enough to pay out depositors at troubled banks. Banks that experience bank runs will liquidate their assets to pay depositors; when they run out of resources they go bankrupt and the DI authority steps in, which pays the remaining bank customers.[9] The DI is incomplete in that everyone is paid an amount c_1, which is promised to those who withdraw early, in period 1. At troubled banks, the late depositors panic and thus get only the amount of c_1 at $t = 1$.[10] We will first consider the social planner's problem for this arrangement and then explain how it can be decentralized.

[9] As in Diamond and Dybvig (1983) and Cooper and Ross (1998), we assume sequential servicing whereby all depositors withdrawing in period $t = 1$ line up in a queue and are paid the promised amount of c_1 until the bank runs out of resources. There is another approach to this, in which troubled banks know at the beginning of $t = 1$ that they will fail, and they pay out every customer a reduced amount so that everyone is paid (and paid the same amnount). See, e.g., Allen and Gale (1998) and Dávila and Goldstein (2023) for this line of modeling.

[10] They then save it until $t = 2$ using the storage technology, at which time they consume the goods.

The social planner's problem for the DI scheme is as follows:[11]

$$\max_{c_1,c_2,i_1,i_2} \Big\{(1-\alpha)\left[\pi\, u(c_1) + (1-\pi)\, u(c_2)\right] + \alpha\, u(c_1)\Big\} \tag{3}$$

s.t.

$$\pi\, c_1 \leq 1 - i_1 - i_2\,, \tag{4}$$

$$(1-\pi)\, c_2 \leq R\, i_1\,, \tag{5}$$

$$c_1 \leq 1 - \tau\, i_1 + \frac{1-\alpha}{\alpha}\, i_2\,. \tag{6}$$

Here c_i denotes the consumption amount promised by a bank to those who withdraw in period $i = 1, 2$; i_1 denotes the bank's investment into the productive technology, and i_2 is the amount the bank contributes to the DI fund. Note that we assume that each bank services a measure one of depositors.

Eq. (3) has two terms. The first represents the utility of depositors at banks with no troubles (their fraction is $1-\alpha$). And the second term represents the utility of depositors at troubled banks (their proportion is α). The second term looks different because all customers of troubled banks consume the same amount, c_1.[12] Constraint (4) refers to the payments to the early depositors at a healthy bank. Since only fraction π of depositors are the early type, the total amount the bank has to pay at $t = 1$ is what remains after it collects deposits, the total amount of which is 1, and invests an amount i_1 into the productive technology and makes a payment i_2 into the DI scheme. Eq. (5) refers to the payments to the late depositors at healthy banks. Eq. (6) refers to the payments to all depositors at a troubled bank. The left-hand side of it can be written as $1 \cdot c_1$, which states that all (i.e., the fraction 1 of) the depositors of the bank withdraw at $t = 1$ and receive c_1. The troubled bank liquidates all its assets and pays c_1 to some depositors until it runs out of all funds, after which it goes bankrupt. The DI scheme pays out the remaining bank depositors. Since the total amount collected from all banks by the DI authority is i_2, the scheme will pay the amount i_2/α to each troubled bank's customers. The total amount paid to the depositors of a troubled bank is $(1-i_1-i_2)+(1-\tau)i_1+i_2/\alpha$, where the first term represents the amount left in the bank after investing into the productive technology and making a contribution into the DI scheme; the second term is the amount received by the bank after liquidating long-term assets; and the third term is the amount paid by the DI scheme. When we collect the like terms we obtain what appears on the right-hand side of (6).

The allocation obtained as the solution to the social planner's problem (3)–(6) can be decentralized as a bank contract. This contract, that we refer to as the DI scheme, does not save the banking system from bank runs. They will happen, but depositor will be protected (to the extent that every depositor claiming to be the early type receives c_1).

[11] All four variables c_1, c_2, i_1, i_2 here and elsewhere are nonnegative. For simplicity, we omit these constraints throughout the paper.

[12] It follows from discussion above that (i) the early type at healthy banks receive c_1 at $t = 1$; (i) all depositors at troubled banks (both the early and late types) receive c_1 at $t = 1$.

The reason for existence of bank runs under the DI regime can be explained by considering, for a moment, a slightly modified version of our model where the deposit insurance authority compensates depositors an amount slightly less than c_1, specifically $c_1 - \epsilon$. The shortfall in compensation, ϵ, could be interpreted in two ways. (i) Depositors might fear a delay in receiving their insured deposits. The discomfort and uncertainty associated with waiting for these payments, especially during financial distress, can act as a psychological burden. This apprehension might prompt depositors to prefer withdrawing their deposits directly from the bank, rather than waiting for the DI authority to process and pay out the insurance claim. (ii) Deposit insurance schemes typically do not cover all types of deposits or amounts fully. For instance, the Federal Deposit Insurance Corporation (FDIC) in the U.S. insures deposits up to a limit of $250,000 per depositor, per insured bank, for each account ownership category. Therefore, ϵ can also represent the uninsured portion of deposits that exceeds the coverage limit. Depositors with amounts above the insured limit might be motivated to withdraw their funds to avoid potential losses above this threshold.

For the purpose of analytical simplicity and to focus on the core dynamics at play, we consider a limiting scenario where ϵ approaches zero. In this theoretical construct, the differential between the insured amount and the full deposit value becomes negligible. However, even this infinitesimally small ϵ can trigger a bank run.

2.3 Privately funded liquidity insurance

The second regime we consider is liquidity insurance (LI). It takes the form of a centralized arrangement whereby participating banks make contributions in period $t = 0$ and make claims in period $t = 1$ if they are in trouble. To access funds from the liquidity insurance fund, a bank must provide collateral. For simplicity, we assume that the bank will use all of its assets for this purpose. These assets will be returned to the banks in period $t = 2$.[13] The amount of collected liquidity will be sufficient to stem any possible panic. Thus we consider the following social planner's problem:

$$\max_{c_1,c_2,i_1,i_2} \left\{\pi\, u(c_1) + (1-\pi)\, u(c_2)\right\} \tag{7}$$

s.t.

$$\pi\, c_1 \leq 1 - i_1 - i_2\,, \tag{8}$$

$$(1-\pi)\, c_2 \leq R\, i_1\,, \tag{9}$$

$$c_1 \leq 1 - i_1 + \frac{1-\alpha}{\alpha}\, i_2\,. \tag{10}$$

Note that unlike in (3), the social planner maximizes $\pi u(c_1) + (1-\pi)u(c_2)$. This is because the LI scheme provides liquidity to banks to the extent that there will be no bank runs and the second

[13] The liquidity insurance regime is somewhat related to interbank lending. Even when loans are secured, the interbank market can freeze and stop functioning as it happened during the 2007-2009 global financial crisis. See Allen et al. (2009) and Acharya et al. (2012) for theoretical analysis as to why the interbank market may loose its efficiency during crises. The secured lending in the LI regime proposed in our paper is an attempt to overcome some difficulties that the interbank market may experience during a financial crisis.

term in (3) disappears. And the term in (6) associated with liquidation costs of long-term assets disappears for the same reason, and as a result we obtain (10) instead. Note also that condition (10) prevents runs since banks will have enough resources to pay c_1, and thus there will be no liquidation of assets during a run. The LI scheme collects presumably larger bank contributions. These additional funds could have been used in the productive activity. This opportunity cost of larger DI premiums is traded-off against the upside of preventing runs. Our LI scheme is partly related to interbank market liquidity insurance (see, e.g., Castiglionesi and Wagner 2013). Interbank lending is one of the most important source of liquidity in the financial system. Unlike the LI scheme in this paper, most interbank lending is carried out in pairwise transactions between two financial institutions. Interbank loans may be both uncollateralized and collateralized; however, during financial crises banks stop using the former and reduce the use of the latter (see, e.g., Engler and Steffen 2016). The LI scheme is an attempt to overcome these type of difficulties during a financial panic.

To maintain analytical tractability in our model, we do not explicitly incorporate moral hazard. Nevertheless, we acknowledge its significance and account for its effects indirectly. Consider for a moment an environment similar to ours but featuring a broader array of productive technologies with varying risk levels (see, e.g., Cooper and Ross 2002). In this scenario, assume that monitoring the performance of a bank's portfolio is costly. Under such conditions, both insurance schemes–DI and LI–would likely induce moral hazard, prompting banks to adopt riskier behaviors than they would in the absence of such schemes.

In the DI framework, all troubled banks would be liquidated, their assets seized by the DI authority, and their depositors would be compensated. In the LI framework, however, troubled banks would be rescued regardless of whether their problems stem from insolvency or mere liquidity issues. This distinction introduces an additional layer of moral hazard in the LI framework compared to the DI framework.

To account for the costs associated with this heightened moral hazard under the LI scheme, we assume that the premiums collected by the LI authority are never returned to the contributing banks. Although not explicitly modeled, one may think of these resources being used to mitigate the consequences of the increased moral hazard, such as covering the heightened losses from troubled banks, managing bank failures, and/or enhancing the monitoring of bank activities. For instance, in the United States, clearinghouses–which serve as a practical example of the LI framework–actively monitored their member banks, auditing their balance sheets and curbing excessive risk-taking (Jaremski, 2015).

2.4 Runs preventing contracts

The third regime we study in this paper is the runs preventing contracts (RPC) considered by Cooper and Ross (1998)[14]. No insurance is involved in this arrangement, and banks handle the issue of bank runs on their own. Each bank offers a contract with a sufficiently low payment to the early type that it has enough resources to stave off a run. The contract promises a payment c_1 to early types so that it satisfies the no-runs condition $c_1 \leq 1 - \tau i$. There will be no runs because a bank will always have enough resources to pay everyone claiming consumption at $t = 1$. The associated social planner's problem for the RPC is formulated as follows:

$$\max_{c_1,c_2,i_1,i_2} \left\{ \pi\, u(c_1) + (1-\pi)\, u(c_2) \right\} \tag{11}$$

s.t.

$$\pi\, c_1 \leq 1 - i_1 - i_2\,, \tag{12}$$

$$(1-\pi)\, c_2 \leq R\, i_1 + i_2\,, \tag{13}$$

$$c_1 \leq 1 - i_1 \tau\,. \tag{14}$$

Unlike in the two insurance problems above, variable i_2 in this problem represents additional liquidity a bank puts aside for the late type in addition to the investment i_1 into the productive technology.[15] Note that this regime, because of the lack of interbank coordination, does not use aggregate information on the fraction of troubled banks, α. As in the case of LI, there is a certain trade-off: additional funds are kept out of productive activity to prevent runs; but the upside is elimination of bank runs.

2.5 Reformulation of the three problems

As in Diamond and Dybvig (1983), the solution allocation from each of the three social planner problems above can be decentralized as bank contracts whereby a bank accepts deposits from agents at $t = 0$ and promises to pay c_1 to those who claim consumption at $t = 1$ and c_2 to those who wait till $t = 2$.

In this paper we compare strengths and weaknesses of the three approaches above, namely: (i) the DI scheme (3)–(6); the LI scheme (7)–(10); and the runs preventing contracts, or RPC, described in (11)–(14). For a given regime, we would like to identify circumstances under which it would be the best choice for society.

To simplify analysis, it will be useful to eliminate variables i_1 and i_2 in these three problems and keep only the two consumption variables. The resulting problems will allow us to use two-dimensional graphical analysis. We first turn to the DI problem (3)–(6). It can be shown that the feasible set of bundles (c_1, c_2) satisfying (4)-(6) is represented by the area under the broken line

[14] We use the same abbreviation, RPC, as Cooper and Ross (1998).

[15] This may be viewed as a form of self insurance.

CAB in Fig. 1. More formally, the problem can be described as follows:[16]

$$\max_{c_1,c_2} \Big\{ \big[\alpha + (1-\alpha)\pi\big]\, u(c_1) + (1-\alpha)(1-\pi)\, u(c_2) \Big\}\,, \tag{15}$$

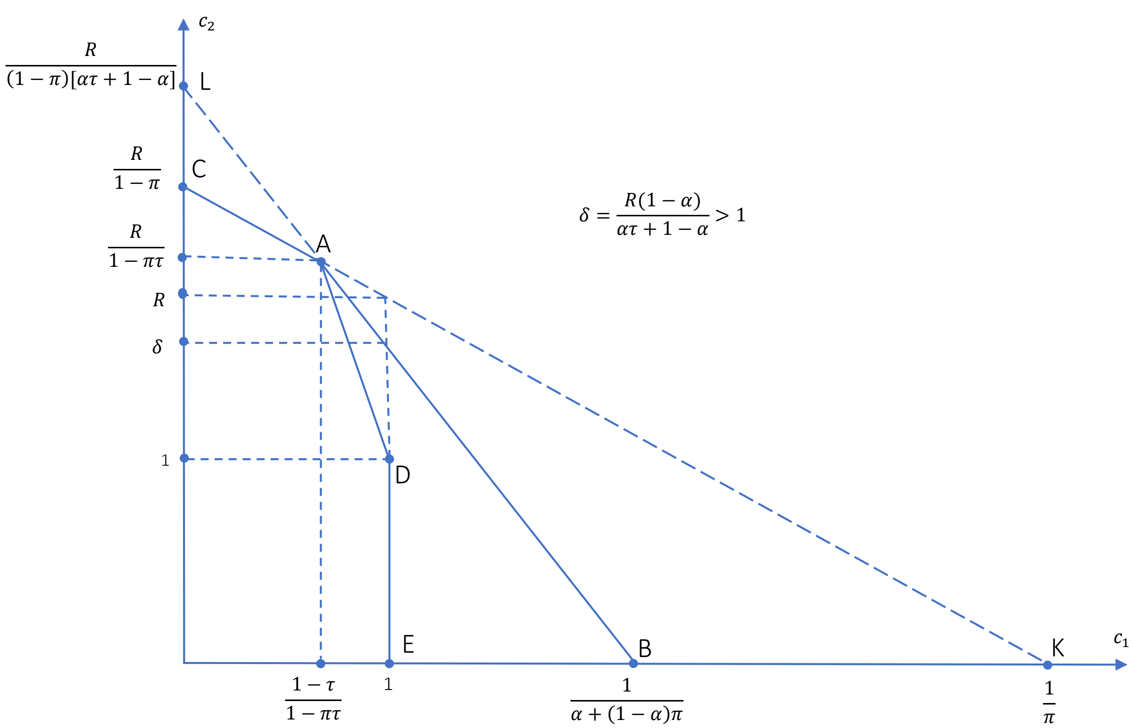
s.t.

$$\big[(1-\alpha)\pi + \alpha\big] c_1 + \frac{1-\pi}{R}\big[\alpha\tau + 1 - \alpha\big] c_2 \leq 1\,, \tag{16}$$

$$\pi c_1 + \frac{1-\pi}{R} c_2 \leq 1\,. \tag{17}$$

(The details are in the proof of Proposition 1 below.) Constraints (16) and (17) are represented in Fig. 1 by segments AB and CA respectively. The coordinates of point $A = ((1-\tau)/(1-\pi\tau), R/(1-\pi\tau))$ are found by jointly solving equations (16) and (17). The coordinates of points B and L can be found by plugging $c_2 = 0$ or $c_1 = 0$ into eq. (16). Similarly, one can use eq. (17) for finding the coordinates of points K and C. It can also be verified that point $(1, R)$ lies on segment CK by plugging its coordinates into (17). We denote the solution to problem (15)-(17) as (c_1^d, c_2^d).

Figure 1: Private DI



Segments AB and CA represent eqs. (16) and (17) respectively.

For some of our results we will need the following assumption.

Assumption 2.

$$R > \frac{\alpha\tau + 1 - \alpha}{1-\alpha}. \tag{18}$$

[16]For convenience, we have rearranged terms in (3).

Let us provide a comment on the assumption. Define

$$\delta = \frac{R(1-\alpha)}{\alpha\tau + 1 - \alpha}\,, \tag{19}$$

where the expression for δ is found by plugging $c_1 = 1$ into (16). Segment AB in Fig. 1 representing constraint (16) goes through point $(1, \delta)$. Assumption 2 is equivalent to $\delta > 1$ and requires that AB lies above AD. It is satisfied when α and/or τ are relatively small (or, alternatively, when R is relatively large).

This assumption will help us narrow down the location of the solution to the DI problem.

Proposition 1. *Let Assumption 2 hold. Then the two constraints* (16) *and* (17) *describe the set of feasible allocations for the social planner's problem* (3)–(6)*. And the solution to this problem satisfies this condition:*

$$1 < c_1^d < c_2^d < \delta\,, \tag{20}$$

where δ is as in (19)*.*

The proposition implies that the solution lies within segment AB in Fig. 1, i.e. the relevant constraint is (16) and not (17). Furthermore, the solution (c_1^d, c_2^d) lies above the 45-degree line going through the origin and point D; and it is below the horizontal line $c_2 = \delta$, where δ is as in (19). In Fig. 2, the solution (c_1^d, c_2^d) should lie between points G and H on segment AB. The proof of the Proposition is found in Appendix A.1.

For the LI problem, the feasible set of (c_1, c_2) bundles that satisfy constraints (8)–(10) can be shown to be represented as the area under line CB in Fig. 2. More formally, this problem can be restated as follows:[17]

$$\max_{c_1, c_2} \Big[\pi u(c_1) + (1-\pi)u(c_2)\Big] \tag{21}$$

s.t.

$$\big[(1-\alpha)\pi + \alpha\big]c_1 + \frac{1-\pi}{R}c_2 \leq 1. \tag{22}$$

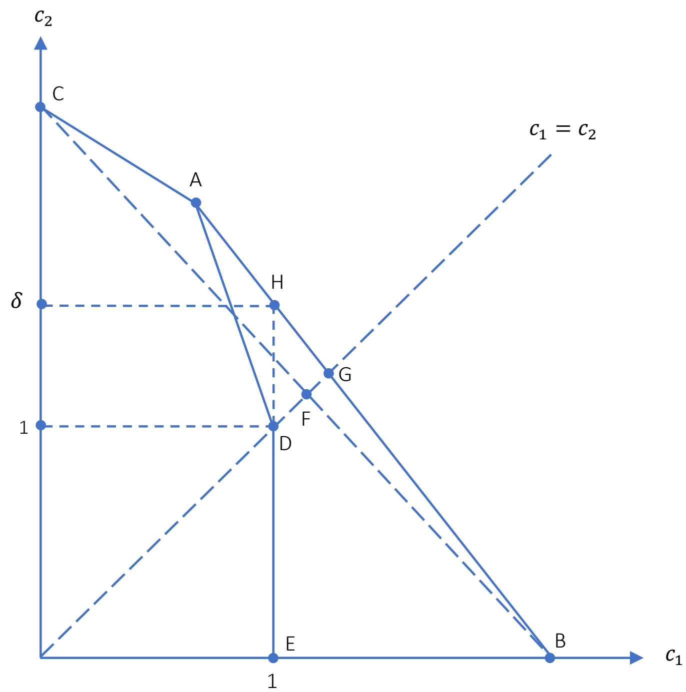
It can be easily seen that the frontier of the feasible set for this problem lies strictly below the frontier for the DI problem (15)–(17), i.e. CB lies below CAB (see Fig. 2). We denote the solution to problem (21)-(22) as (c_1^l, c_2^l).[18]

And finally, the feasible set of bundles (c_1, c_2) for the RPC problem can be described by three

[17] We show in Appendix A.2 that all three constraints (8)–(10) must bind. Then we obtain constraint (22) by simply eliminating i_1 and i_2.

[18] One can narrow down the location of (c_1^l, c_2^l) on segment CB. More precisely, it can be shown that $\frac{\pi}{\pi + (1-\pi)\alpha} < c_1^l < c_2^l < R$.

Figure 2: DI, LI and RPC



The solution to the DI problem lies between points G and H on segment AB. The frontier of the feasible set for the liquidity insurance problem lies strictly below the frontier for the DI problem, i.e. CB lies below AB.

new constraints; and the problem itself can be restated as follows:[19]

$$\max_{c_1,c_2} \Big[\pi u(c_1) + (1-\pi)u(c_2)\Big] \tag{23}$$

s.t.

$$\left[\pi + \frac{R-1}{\tau}\right] c_1 + (1-\pi)c_2 \leq 1 + \frac{R-1}{\tau}\,, \tag{24}$$

$$\pi c_1 + \frac{1-\pi}{R} c_2 \leq 1\,, \qquad c_1 \leq 1\,. \tag{25}$$

Let the solution to this problem be denoted by (c_1^n, c_2^n).

The feasible set satisfying constraints (24)–(25) is shown in Fig. 2 as the area under the broken line $CADE$. In particular, constraint (24) represents segment AD whereas constraints (25) represent segments CA and DE. Note that this set is a subset of the feasible set for the DI problem, which lies below the broken line CAB. It will be useful to remember the following on the actual location of the solutions to the RPC problem and the DI problem: (i) as Cooper and Ross (1998) demonstrate, the relevant constraint for the RPC problem is segment AD (the solution is always inside AD or at point A); in other words, eq. (24) is the relevant constraint; (ii) our Proposition 1 states that the solution to the DI problem lies strictly within AB (and more precisely, between

[19]See Appendix A.3 for derivations.

points G and H).

3 Comparison of the three regimes

Let V_d denote the optimal value of the expected utility in the DI problem (15)–(17). Similarly, let V_l denote the optimal value of the expected utility in the LI problem (21)–(22) and V_n the optimal value in the RPC problem (23)–(25).

3.1 The role of the proportion of troubled banks α

We will argue in this section that when the probability of bank runs, α, is small, both insurance arrangements are socially preferable to the RPC. It is quite reasonable to assume that α is small. For example, in many OECD countries, the ratio of the size of DI fund to covered deposits is less than 1%.[20] Dávila and Goldstein (2023) discuss in some detail various estimates of historical bank failure probabilities and choose the value of 2.5% in their quantitative analysis.

We will need the following critical value for α:

$$\alpha^* = \frac{R-1}{R-1+\tau}. \tag{26}$$

This is the value of α at which segment AB in Fig. 1 goes through point D and thus the relevant constraint lines for the DI and RPC problems will coincide.[21] Note that in this extreme case $\delta = 1$, where δ is as in (19). For given values of R and τ, Assumption 2 holds for $\alpha \in [0, \alpha^*)$, and it does not hold for $\alpha \in [\alpha^*, 1]$.

Proposition 2. *There exist threshold values $\hat{\alpha}_i \in (0, \alpha^*)$ such that $V_i > V_n$ for $\alpha < \hat{\alpha}_i$ and $V_n > V_i$ for $\alpha > \hat{\alpha}_i$, where $i = d, l$.*

The proposition states that the DI scheme delivers a better deal than the RPC when the probability of bank runs α is small (when $\alpha < \hat{\alpha}_d$), and the reverse is true for large values of α (when $\alpha > \hat{\alpha}_d$). A similar threshold level, α_l, exists for the LI problem. Intuitively, the DI and LI insurance premiums increase as α goes up, and at some point the RPC, which may be viewed as self-insurance, becomes a more attractive option.

Note that V_n does not depend on α at all. The idea of the proof for the DI case is to demonstrate that V_d is a decreasing function of α and show that at the two extremes we have opposite inequalities: $V_d > V_n$ when $\alpha = 0$ and $V_n > V_d$ when $\alpha = \alpha^*$. A similar argument applies to the case of the LI scheme. A detailed proof is in the Appendix.

Both insurance problems yield a higher expected utility than the RPC when α is small. Intuitively, it can be easily seen from Fig. 1. As α approaches 0, point B approaches point K; then

[20] E.g., it is 0.32% in Belgium and Canada, 0.73% in Denmark, 0.21% in France and 0.37% in Germany (see Demirgüç-Kunt et al. 2015).

[21] We will show that in this case both solutions lie within segment AD and thus segment DB is irrelevant for the DI problem.

both segments AB and CB (the relevant constraints for the DI and LI problems respectively) move away from segment AD and approach segment CK represented by the resource constraint (17). Proposition 2 states that $V_d > V_n$ in this case. And also note that $V_l = V_d$ when $\alpha = 0$ since both the expected utility and constraints are the same for both problems. Therefore, for small values of α, we have both $V_d > V_n$ and $V_l > V_n$. In other words, it is best to have an insurance scheme when α is small. However, we are unable to say anything definite about the comparison between V_d and V_l for the case of small α. The complication arises from the fact mentioned above: $V_l = V_d$ when $\alpha = 0$. Later in section 4, for the CRRA family of utility functions, we will be able to demonstrate that $V_d > V_l$ when both α and τ are small (see Proposition 6). Our numerical analysis in subsection 4.2 extends this result to the case when α is moderate and τ is not excessively large.

3.2 The role of the liquidation cost τ

Now let us explore the role of the liquidation cost τ. We can establish some results at the two extreme values of τ: when it is 0 and when it is 1.

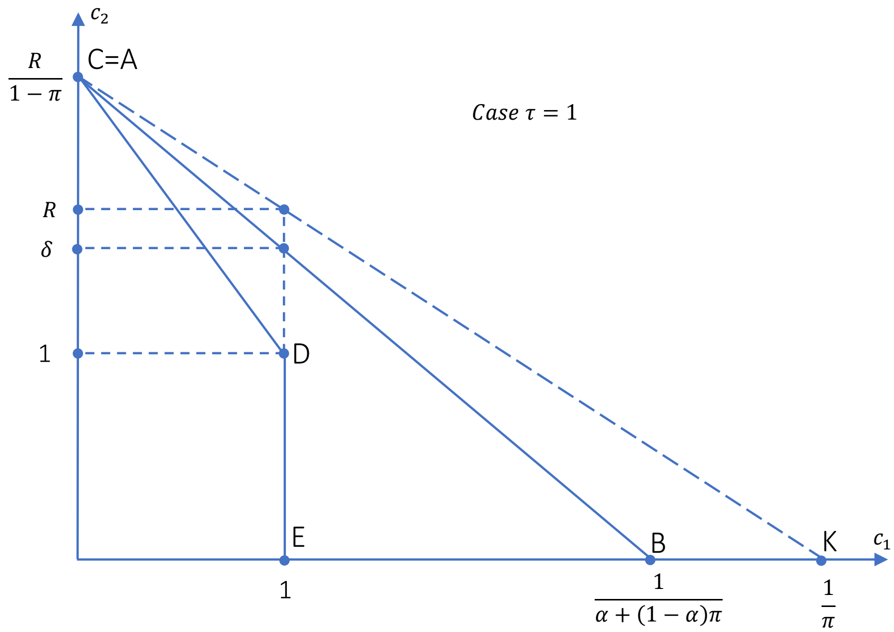
Proposition 3. *Let Assumption 2 hold for $\tau = 1$. If the liquidation cost $\tau = 1$ or near 1, then V_l is larger than both V_d and V_n.*

Thus, in this case the LI scheme provides the best solution among the three regimes. Let us understand the intuition behind this result. There are two forces to keep in mind when comparing V_l and V_d. On the one hand, there are liquidation costs that some banks have to bear in the DI case (and there are none in the case of the LI scheme); on the other, banks pay lower insurance premiums than they do under the LI scheme. When liquidation costs are high, the former effect outweighs the latter, which results in $V_l > V_d$. More technically, when $\tau = 1$, point A coincides with C and the relevant constraints for the DI and LI problems are the same, namely segment $AB = CB$ (see Fig. 3). Note that the LI problem and DI problem maximize the expected utility functions $U = \pi\, u(c_1) + (1-\pi)\, u(c_2)$ and $U^d = [\alpha + (1-\alpha)\,\pi]\, u(c_1) + (1-\pi)(1-\alpha)\, u(c_2)$ respectively, and that U puts more weight on the utility of the patient type, $u(c_2)$ than U_d does. Since the solution to the DI problem (c_1^d, c_2^d) exhibits $c_1^d < c_2^d$, we have $V^l > V^d$.

To understand the intuition behind $V_l > V_n$, note that V_l does not depend on the liquidation cost (τ is not present in (7)–(10)) whereas V_n does. As liquidation costs increase, banks in the RPC regime have to decrease consumption of the early type, c_1, in order to maintain the no-runs condition $c_1 < 1 - \tau i$. In the limiting case $\tau = 1$, this adjustment becomes too large. In pursuing the RPC, banks engage in self-insurance and (collectively) do not take advantage of the fact that only a fraction of banks experience runs. In contrast, under the LI scheme, the aggregate information on α is taken into account but no adjustment to allocation takes place as τ changes.

Note that the above comparison between V_l and V_n reveals a very important point: V_l depends on α but not on τ whereas V_n does depend on τ but not on α. These are relative strengths and

Figure 3: The Role of Liquidation Costs I



When the liquidation cost $\tau = 1$, point A coincides with C and the relevant constraints for the DI and LI problems are the same, namely segment $AB = CB$.

weaknesses of the LI scheme and the RPC. Finally note that V_d depends on both α and τ. Thus, unlike in the other two problems, the social planner in problem (3)–(6) takes into account both the information on the fraction of failing banks throughout the economy, α, and liquidation costs, τ, and strikes some balance between relative strengths and weaknesses of the other two approaches.

Now let us turn to the other extreme and assume that the liquidation cost $\tau = 0$, i.e. at $t = 1$ it is costless to get back any amount of investment into the productive technology made in period $t = 0$. In this case, the upper bound for the thethreshold levels $\hat{\alpha}_d$ and $\hat{\alpha}_l$ in Proposition 2 found in (26) yields too large a number: $\alpha^* = 1$. We can provide a tighter upper bound:[22]

$$\bar{\alpha} = \frac{\pi[u'(1) - Ru'(R)]}{\pi u'(1) + (1-\pi)Ru'(R)}. \tag{27}$$

Note that $\bar{\alpha} > 0$ because, as DD show, under Assumption 1, $u'(1) - Ru'(R) > 0$. And it is clear that $\bar{\alpha} < 1$.

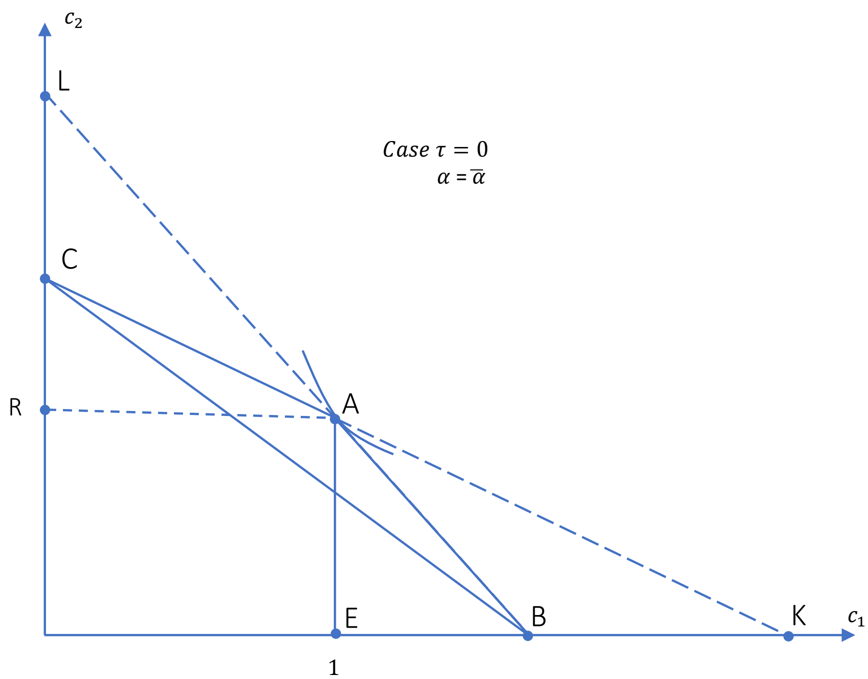
Proposition 4. *Suppose the liquidation cost $\tau = 0$ or near 0 and $\alpha \geq \bar{\alpha}$. Then V_n is larger than both V_d and V_l.*

The intuition behind this result is as follows. When $\tau = 0$, there is no cost of liquidation that banks under the RPC would otherwise bear. And when the probability of bank failure α is

[22]See Appendix A.6 for details on how to obtain $\bar{\alpha}$.

large, i.e. $\alpha > \bar{\alpha}$ where $\bar{\alpha}$ is explained below, the cost of contribution by banks to the common fund under the other two regimes outweighs the benefits when compared with the RPC. More technically, let U and U^d denote the expected utility functions $U = \pi\, u(c_1) + (1-\pi)\, u(c_2)$ and $U^d = [\alpha + (1-\alpha)\,\pi]\, u(c_1) + (1-\pi)(1-\alpha)\, u(c_2)$ and let F^d denote the feasible set for the DI problem. If U^* and U_d^* denote the optimal utility levels obtained at the solutions to the maximization of U subject to F^d and maximization of U_d subject to F^d respectively, we can claim that $U^* > U_d^*$. This is because $u(c_1^d) < u(c_2^d)$ according to Proposition 1, and U puts more weight on the utility of the late type, $u(c_2)$, than U_d does. The slope $\bar{\alpha}$ is such that the indifference curve of U is tangent to segment AB in Fig. 4 at point $A = (1, R)$. For levels of α greater than $\bar{\alpha}$, point A will be a corner solution to the problem $\max U$ s.t. F_d. And since point A is common to both F_d and the feasible set of the RPC problem, we conclude that $V_n > V_d$. A similar argument can be used for proof of $V_n > V_d$. A detailed proof is found in the Appendix.

Figure 4: The Role of Liquidation Costs II



When the liquidation cost $\tau = 0$ and $\alpha = \bar{\alpha}$, the indifference curve of U is tangent to AB at point A.

4 The CRRA utility function

In this section we consider a specific, the constant relative risk aversion (CRRA), family of utility functions:

$$u(c) = \frac{c^{1-\eta}}{1-\eta}, \qquad \eta > 1\,. \tag{28}$$

(Note that since $\eta > 1$, Assumption 1 is satisfied.) This specification allows us to obtain results that go beyond what we were able to do in the general case in Section 3.

4.1 Cases when DI is dominant

We now consider two special cases when the DI scheme is superior to the LI and RPC regimes.

Proposition 5. *Suppose $\tau < 1$. Let Assumption 2 hold. Then V_d is larger than both V_n and V_l if η is large enough, i.e. there exists $\hat{\eta}$ such that $V_d(\eta) > V_n(\eta)$ and $V_d(\eta) > V_l(\eta)$ for $\eta > \hat{\eta}$.*[23]

Thus, when agents are very risk averse, the DI is socially preferable over both the LI scheme and the RPC. The intuition behind this result is as follows. As agents become more risk averse, i.e. as η increases, they want more consumption smoothing across the two possible type realizations. Thus, in the limit, as $\eta \to \infty$, the solution to any of the three social planner's problems exhibits the same feature: $c_1 = c_2$. Thus, in this limiting case, the solution for each problem is located at the intersection of the 45-degree line and the relevant constraint. In Fig. 2, points G, F and D represent the solutions to the DI, LI and RPC problems respectively. Because segment AB is always located above segment CB and point D, we conclude that point G will always be located strictly above points F and D. Thus, the same conclusion can be made for large enough values of η. In other words, for large enough η, we have both c_1^d and c_2^d being greater than c_1^i and c_2^i for $i = l, n$. Since each V_i is a weighted average of $u(c_1)$ and $u(c_2)$, we conclude that V_d is greater than both V_l and V_n.

From the above, the reason for the dominance of the DI regime is that the relevant constraint for the DI problem, segment AB, lies above the relevant constraints for the LI and RPC problems, segments CB and AD respectively. And that fact follows from the inherent advantage of the DI scheme: the collective amount of premiums collected, i.e. the size of the DI fund, is smaller than that in the LI scheme and the collective amount of liquidity banks put aside for self-insurance purposes in the RPC case. Thus, the banking system under the DI regime puts more resources into the productive technology.

Proposition 6. *Suppose*

$$\frac{1}{1-\alpha} < R \le 2. \tag{29}$$

Then $V_d > V_l$ for small values of α and τ.

[23] In fact, the assumption $\tau < 1$ is not needed for $V_d(\eta) > V_n(\eta)$.

Proposition 2 established that when the probability of bank runs, α, is low, both V_d and V_l are greater than V_n. However, it says nothing as to which of the two regimes, the DI or LI, dominates the other. The comparison is tough due to the fact that $V_d = V_l$ when $\alpha = 0$. Proposition 6 fills this gap. The the proof of Proposition 6 inspects the derivatives of $V_d(\alpha)$ and $V_l(\alpha)$ with respect to α at $\alpha = 0$. It turns out that when condition (29) holds, we can establish that $V_d'(0) > V_l'(0)$ when $\tau = 0$. This means that $V_d > V_l$ for small positive values of α. By continuity, the same is true for small positive values of τ and α.

Thus, for a CRRA utility function, there are two cases when V_d is larger than V_l and V_n:

(i) when both α and τ are small. Indeed, from Proposition 2 it follows that when the probability of runs α is small, the DI dominates the RPC. And it follows from Proposition 6 that the DI dominates the LI when the probability of runs α and liquidation cost τ are small.

(ii) when α is small and η is large. Here again, the inequality $V_d > V_n$ follows from Proposition 2. And Proposition 5 implies the inequality $V_d > V_l$.

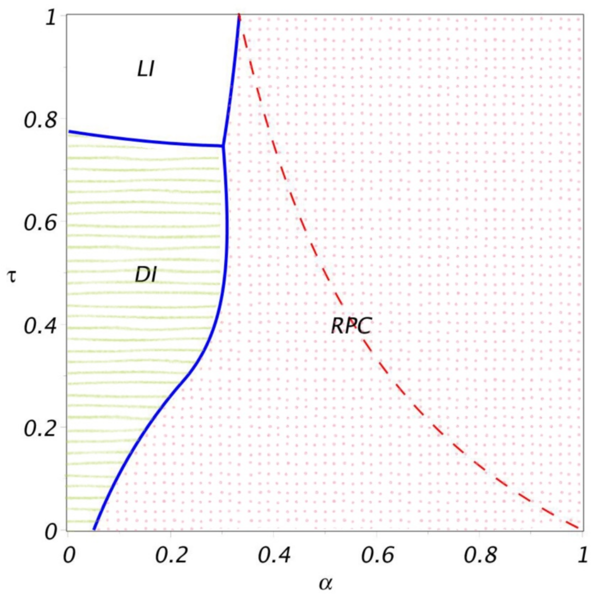
4.2 Numerical analysis

Our numerical analysis reveals more than the theoretical results as many of our propositions assume sufficient, but not 'necessary and sufficient', conditions for the results. For example, $V_d > V_l$ can hold even when condition (29) does not hold. The role of the liquidation cost τ and the probability of bank runs α as well as their interplay with the remaining parameters can be seen in Fig. 5.

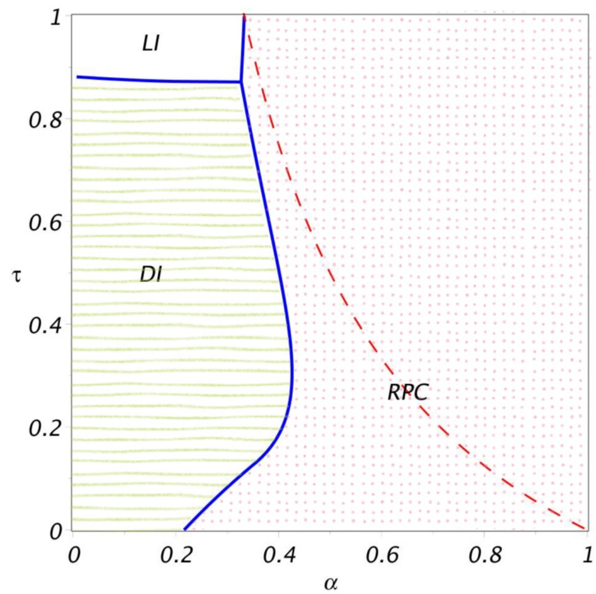
The value of π, the proportion of early consumers, is 0.5 throughout the figure. This is consistent with other studies that use versions of the Diamond and Dybvig (1983) model. For example, Li (2017) considers several values of π ranging between 0.2 and 0.85; and Sultanum (2014) uses a symmetric distribution of values for π centered around 0.5. We consider four possible combinations of the remaining parameters η and R. We pick two values for the rate of return R, 1.5 and 1.2, which are the values chosen by Li (2017) and Sultanum (2014) respectively. And we pick two values for the coefficient of relative risk aversion η: 2.0 and 4.0. Elminejad et al. (2022) analyze 92 studies and report that common calibration values for η vary between 2.5 and 10 but other values such as 1 and 20 appear often too. Li (2017) uses the values of 3 and 8, and Sultanum (2014) uses 3.

The coefficient of relative risk aversion η equals 2.0 in the left two diagrams and 4.0 in the right two diagrams. Similarly, the rate of return R equals 1.5 in the top two diagrams and 1.2 in the bottom two. In each of the four diagrams, each of the three areas represents combinations of α and τ for which a particular regime dominates the other two. As one can easily see, for small values of α and τ the DI scheme is the dominant regime. And the RPC dominates when α is large. When α is relatively small and τ is large, the LI scheme is the dominant regime. An increase in η pushes up the boundary between the DI and LI dominance areas. This is consistent with Proposition 5, according to which, when other parameters are fixed, the DI starts dominating the LI as $\eta \to \infty$.

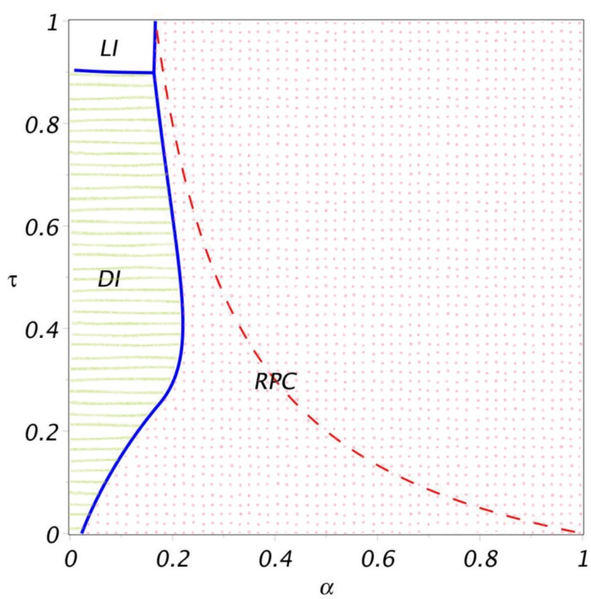
Figure 5: CRRA utility function $u(c) = \frac{c^{1-\eta}}{1-\eta}$



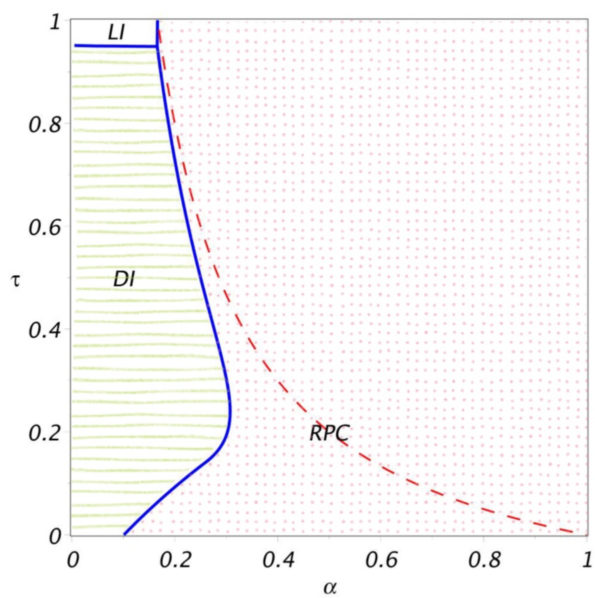
(a) $\pi = 0.5, \eta = 2.0, R = 1.5$



(b) $\pi = 0.5, \eta = 4.0, R = 1.5$



(c) $\pi = 0.5, \eta = 2.0, R = 1.2$



(d) $\pi = 0.5, \eta = 4.0, R = 1.2$

Each of the three areas represents dominance of a particular regime. Assumption 2 holds in the area to the left of the dashed line.

And finally, the effect of an increase in R is a bit more complex; it pushes the boundary between the DI and RPC dominance areas to the right; it also pushes the boundary between the DI and LI dominance areas down.

The dashed line represents all combinations of (α, τ) that satisfy $R = (\alpha\tau + 1 - \alpha)/(1 - \alpha)$ for a given value of R (where $R = 1.5$ or $R = 1.2$). Thus, all (α, τ)-combinations to the left of that line satisfy condition (18), i.e. Assumption 2. For given values of R and τ, the value of α of the corresponding point on the dashed line is computed as α^* in eq. (26). Let us consider, for example, diagram (a) in Fig. 6, where $R = 1.5$, and pick $\tau = 0.4$. Then the corresponding point on the dashed line is $(0.556, 0.4)$, where the value $\alpha = 0.556$ is computed as α^* in eq. (26). The corresponding point on the solid line separating the DI and RPC areas is $(0.286, 0.4)$, where the value $\alpha = 0.286$ is $\hat{\alpha}_d$ in Proposition 2. And as is stated in Proposition 2, indeed $\hat{\alpha}_d \in (0, \alpha^*)$ since $0 < 0.286 < 0.556$. Alternatively, if we pick $\tau = 0.9$, the corresponding point on the dashed line is $(0.357, 0.9)$; here $\alpha^* = 0.357$ is found using eq. (26). The corresponding point on the solid line separating the LI and RPC areas is $(0.323, 0.9)$, where the value $\alpha = 0.323$ is $\hat{\alpha}_l$ in Proposition 2. And as is stated in Proposition 2, indeed $\hat{\alpha}_l \in (0, \alpha^*)$ since $0 < 0.323 < 0.357$.

According to FDIC (2024), the proportion of insured deposits fluctuated between 54% and 61% from 2013 to 2023, with an average of approximately 58.9%.[24] If we interpret the parameter π in our model as the proportion of insured deposits, it is prudent to consider values of π ranging from 0.5 to 0.6 in our analysis. To this end, Fig. 6 presents the results of our numerical exercise using $\pi = 0.6$, which also serves as a robustness check. As evident from the comparison between Figures 5 and 6, the change in results is minimal, suggesting that our findings are robust to variations in the proportion of insured deposits within this range.

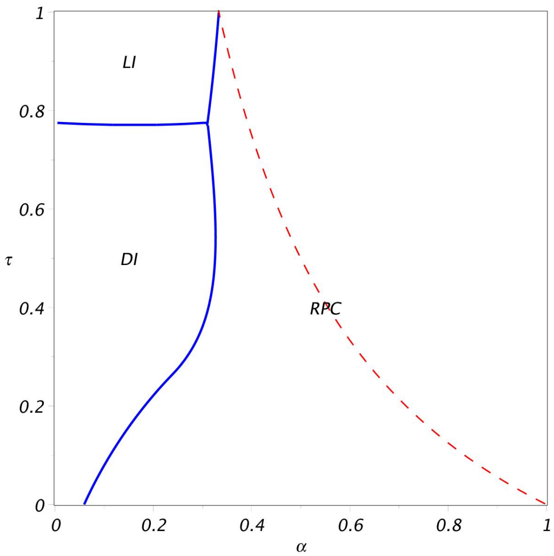
We would like to conclude this section by mentioning values for the probability of runs α and cost of liquidation τ used in two recent studies. Dávila and Goldstein (2023) use $\alpha = 0.025$. Granja et al. (2017) report the average loss incurred by FDIC from selling a failed bank to be 28%, which can be used as the value of the liquidation cost τ. A quick inspection of Fig. 5 leads us to a conclusion that the combination $(\alpha, \tau) = (0.025, 0.28)$ lies in the DI dominance area in all four cases. This finding is robust to the choice of the other three parameters, π, η and R.

5 Extensions

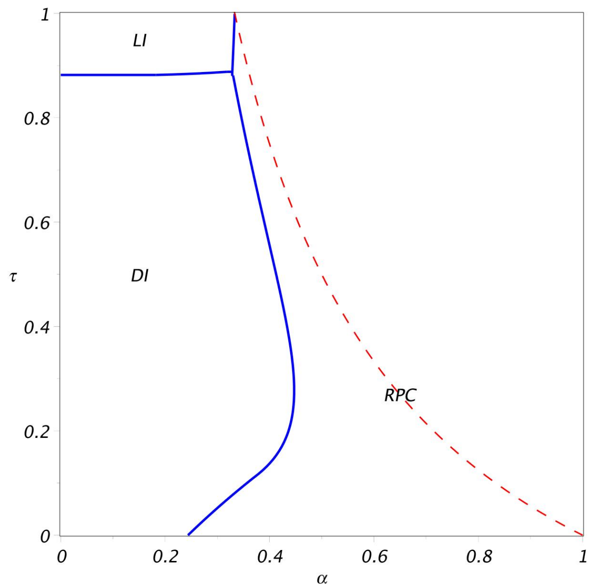
We will present two extensions in this sections.

[24] In response to the global financial crisis of 2007-2009, FDIC implemented the Transaction Account Guarantee Program. This program was initially in effect from October 2008 to December 2010 and was subsequently extended through December 2012 by the Dodd-Frank Act. Upon the program's expiration in January 2013, there was a notable increase in the proportion of uninsured deposits. (See, e.g., Bao et al. 2015.) This makes 2013 a logical starting point, as it marks the return to more typical deposit insurance conditions.

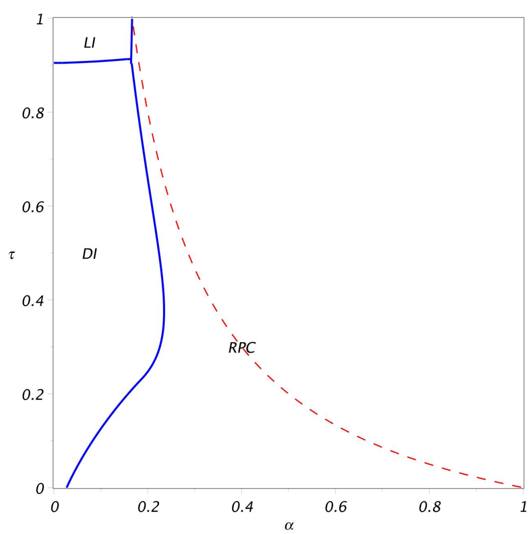
Figure 6: CRRA utility function $u(c) = \frac{c^{1-\eta}}{1-\eta}$



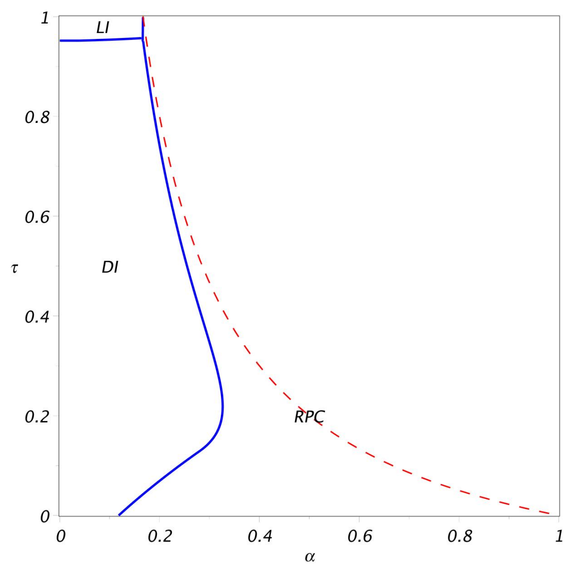
(a) $\pi = 0.6, \eta = 2.0, R = 1.5$



(b) $\pi = 0.6, \eta = 4.0, R = 1.5$



(c) $\pi = 0.6, \eta = 2.0, R = 1.2$



(d) $\pi = 0.6, \eta = 4.0, R = 1.2$

Each of the three areas represents dominance of a particular regime. Assumption 2 holds in the area to the left of the dashed line.

5.1 Government intervention

As mentioned in the introduction, most deposit insurance schemes are privately funded and maintain an explicit ex-ante fund. Our model assumes that such a fund is sufficient to cover all potential losses the banking system may incur. However, if an unusually large shock hits the financial system, a DI fund may exhaust its resources. For instance, the DI fund balance of the Federal Deposit Insurance Corporation (FDIC) in the U.S. went negative twice in its history: during the savings and loan crisis of the late 1980s and early 1990s, and amid the global financial crisis of 2007-2009 (see, e.g., Chart 2 in Ellis 2013 and Fig. 5.1 in FDIC 2017). It appears that the most catastrophic risks in the financial industry are ultimately covered by the government. For example, the total cost of the savings and loan crisis to taxpayers amounted to $132 billion (see Table 3 in GAO 1996). In this section, we explore the possibility of government intervention when the insurance fund is depleted.

We consider a scenario where the fraction of banks experiencing a run unexpectedly exceeds α, the level the DI authority is prepared to cover. In such a case, government intervention becomes necessary, and we assume that taxation is employed to ensure every depositor withdrawing at $t = 1$ receives c_1^d before tax.[25] We will also examine government intervention in the case of liquidity insurance (LI). Notably, no intervention is required in the run-proof contract (RPC) regime, as its arrangement does not depend on α. We will then compare the performance of these three regimes.

Consider a zero-probability event (from the ex-anter perspective) in which the actual fraction of troubled banks is $\tilde{\alpha} = \kappa\alpha$ whith $\kappa > 1$; if $\kappa\alpha > 1$, then we set $\tilde{\alpha} = 1$. [26] In this scenario, the DI authority will be unable to fulfill its obligations and fall short of providing every depositor of every troubled bank the promised amount c_1^d. The LI authority in the case of liquidity insurance will face a similar challenge.

The DI case

Let us examine the events in detail to understand the consumption profile of agents. At time $t = 1$, troubled banks pay out c_1^d to depositors until exhausting their resources. Subsequently, the DI authority services them sequentially: their depositors will get c_1^d from the DI fund. And when the DI fund is exhausted, the realization of $\tilde{\alpha}$ is revealed and the government intervenes. It applies a proportional tax on depositors who receive a positive amount from their bank. The tax collection should be just sufficient to pay the amount $\tilde{c_1}^d$, which is the after-tax consumption of those who collected c_1^d from their bank, to those who received nothing.

[25] Reminder: (c_1^d, c_2^d) and (c_1^l, c_2^l) denote the solutions to the DI and LI problems in the main case, i.e. the problems (15)-(17) and (21)-(22) respectively.

[26] The assumption that the ex-ante probability of such an event is zero implies that this event is unexpected. Therefore, the behavior of agents in this economy up to this zero-probability event will be according to our previous analysis above. Another way to think about a zero-probability event is to imagine a small-probability event, and then let that probability go to zero. For game-theoretic considerations of zero-probability events see, e.g., Myerson (1986) and citations therein.

To determine the tax rate and after-tax consumption levels, we need to find the pre-tax consumption profile of depositors. Before the government taxation, there will be three groups of depositors with different amounts received from banks: (i) zero; (ii) c_1^d; and (iii) c_2^d (promised to be paid at $t = 2$). Let us calculate the number of depositors in each of these three groups by examining three categories of banks:

(a) Healthy banks, numbering $1 - \tilde{\alpha}$. Their depositors consist of

- $(1 - \tilde{\alpha})\pi$ customers (early type) who receive c_1^d;
- $(1 - \tilde{\alpha})(1 - \pi)$ customers (late type) who receive c_2^d.

(b) "Lucky" troubled banks whose customers are serviced by the DI authority; there are α of them. All their customers receive c_1^d.

(c) "Unlucky" troubled banks whose customers are not serviced by the DI authority; their number is $\tilde{\alpha} - \alpha$. Each such bank services $x_1^d < 1$ customers before running out of resources (x_1^d to be determined below). Their depositors consist of

- $x_1^d \cdot (\tilde{\alpha} - \alpha)$ customers who receive c_1^d;
- $(1 - x_1^d) \cdot (\tilde{\alpha} - \alpha)$ customers who receive nothing.

Thus, we have the following numbers for the three groups of depositors:

(i) $N_1^d = (1 - x_1^d) \cdot (\tilde{\alpha} - \alpha)$ depositors receive nothing;

(ii) $N_2^d = [(1 - \tilde{\alpha})\pi + \alpha + x_1^d \cdot (\tilde{\alpha} - \alpha)]$ depositors receive c_1^d;

(iii) $N_3^d = [(1 - \tilde{\alpha})(1 - \pi)]$ depositors receive c_2^d.

To determine the proportional tax rate, μ^d, we have two equations:

$$\tilde{c_1}^d = (1 - \mu^d) \cdot c_1^d \,, \tag{30}$$

$$N_1^d \cdot \tilde{c_1}^d = N_2^d \cdot \mu^d \cdot c_1^d + N_3^d \cdot \mu^d \cdot c_2^d \cdot (1 - \tau) \,. \tag{31}$$

Eq. (30) shows the after-tax consumption for those who received c_1^d. Eq. (31) states that the aggregate after-tax payment to those who received zero should equal the tax collections from groups (ii) and (iii) of depositors who received the before-tax amounts of c_1^d and c_2^d respectively. The tax payment of the late type at healthy banks is implemented through liquidation of assets by their banks as taxes are collected at $t = 1$ and the late types receive their payments from banks at $t = 2$. The multiplier $(1 - \tau)$ in (31) reflects this fact.

From these two equations we can derive the tax rate μ^d and after-tax consumptions:

$$\mu^d = \frac{N_1^d c_1^d}{N_1^d c_1^d + N_2^d c_1^d + N_3^d c_2^d (1 - \tau)} \,, \tag{32}$$

$$\tilde{c_1}^d = (1 - \mu^d) \cdot c_1^d \,, \qquad \tilde{c_2}^d = (1 - \mu^d) \cdot c_2^d \,. \tag{33}$$

Finally, let us determine x_1^d, the number of depositors at unlucky banks who receive c_1^d. Each unlucky bank has the following resources to pay out before failure: $1 - i_1^d - i_2^d$ of liquid assets and $i_1^d(1-\tau)$ received after liquidating its long-term assets (see eqs. (4)-(6)). Together, this amounts to $1 - \tau i_1^d - i_2^d$. Since each of the first x_1^d depositors at such banks receives c_1^d, we have

$$x_1^d = \frac{1 - \tau i_1^d - i_2^d}{c_1^d}\,. \tag{34}$$

Let us show that both i_1^d and i_2^d are functions of (c_1^d, c_2^d). From the proof of Proposition 1 in Appendix A.1 we know that both (4) and (5) bind. Solving for i_1 and i_2 yields

$$i_1^d = \frac{(1-\pi)c_2^d}{R}\,, \qquad i_2^d = 1 - \frac{(1-\pi)c_2^d}{R} - \pi c_1^d\,. \tag{35}$$

From eqs. (34) and (35) we conclude that x_1^d is a function of c_1^d and c_2^d: $x_1^d = x_1^d(c_1^d, c_2^d)$. Therefore, the tax rate μ^d and after-tax consumptions $\tilde{c_1}^d$ and $\tilde{c_2}^d$ as well as N_1^d, N_2^d, N_3^d are functions of (c_1^d, c_2^d). This allows us to compute the welfare level $\tilde{V}_d$ associated with this case, i.e. when DI is combined with government backstop in the form of taxation:

$$\tilde{V}_d(c_1^d, c_2^d) = (N_1^d + N_2^d) \cdot u(\tilde{c_1}^d) + N_3 \cdot u(\tilde{c_2}^d)\,. \tag{36}$$

The LI case

At time $t = 1$, troubled banks pay out c_1^l to depositors the amount until exhausting their resources. They are then serviced by the LI authority sequentially: each troubled bank receives the promised amount from the LI fund until the fund is exhausted. The troubled banks that received resources from the LI authority will pay their depositors the promised amounts. Once the LI fund runs out of resources, the realization of $\tilde{\alpha}$ is revealed, and the government intervenes. It applies a proportional tax on depositors who receive a positive amount from their bank. The tax collection should be just sufficient to pay the amount $\tilde{c_1}^l$, which is the after-tax consumption of those who collected c_1^l from their bank, to those who received nothing.

Although the computation of the tax rate and consumption levels in this case is similar to the DI regime, a small difference leads to somewhat different calculations. First, let us note that the depositors of "lucky" troubled banks will receive the same amounts as those at healthy banks. Each lucky bank receives the promised amount from the LI fund and thus does not have to liquidate long-term assets. This allows the bank to pay the early types the amount c_1^l at $t = 1$ and the late types the amount c_2^l at $t = 2$. The outcomes at the unlucky troubled and healthy banks are similar to those in the DI case.

The formulae for the numbers of depositors in the three groups (those receiving before tax the amounts of zero, c_1^l and c_2^l) are as follows:

(i) $N_1^l = (1 - x_1^l) \cdot (\tilde{\alpha} - \alpha)$ depositors receive nothing;

(ii) $N_2^l = [(1-\tilde{\alpha})\pi + \alpha\pi + x_1^d \cdot (\tilde{\alpha} - \alpha)]$ depositors receive c_1^l;

(iii) $N_3^l = [\alpha(1-\pi) + (1-\tilde{\alpha})(1-\pi)]$ depositors receive c_2^l.

The remaining expressions for $\mu^l, \tilde{c_1}^l, \tilde{c_2}^l, x_1^l, i_1^l, i_2^l$ are identical to those in the DI case, except for the superscripts. In particular, we have the following formulae for μ^l, the proportional tax rate, and x_1^l, the number of depositors at unlucky banks who receive c_1^l:

$$\mu^l = \frac{N_1^l c_1^l}{N_1^l c_1^l + N_2^l c_1^l + N_3^l c_2^l (1-\tau)}, \tag{37}$$

$$x_1^l = \frac{1 - \tau i_1^l - i_2^l}{c_1^l}. \tag{38}$$

Eqs. (37) and (38) reveal a crucial distinction between the LI case under the main scenario (where the proportion of troubled banks is α, as expected) and the case when that proportion is unexpectedly large ($\tilde{\alpha}$). In the former, banks do not liquidate assets, whereas in the latter, the unlucky troubled banks do liquidate all of their long-term assets to pay c_1^l to their depositors (as reflected in (38)). Moreover, all other banks liquidate part of their long-term assets due to government taxation of their late-type depositors (as shown in (37)). These liquidations result in a welfare loss that was not present in the LI case in the main scenario.

This analysis leads us to the following expression for the welfare level $\tilde{V}_l$ associated with the case when LI is combined with a government backstop in the form of taxation:

$$\tilde{V}_l(c_1^l, c_2^l) = (N_1^l + N_2^l) \cdot u(\tilde{c_1}^l) + N_3 \cdot u(\tilde{c_2}^l). \tag{39}$$

The RPC case

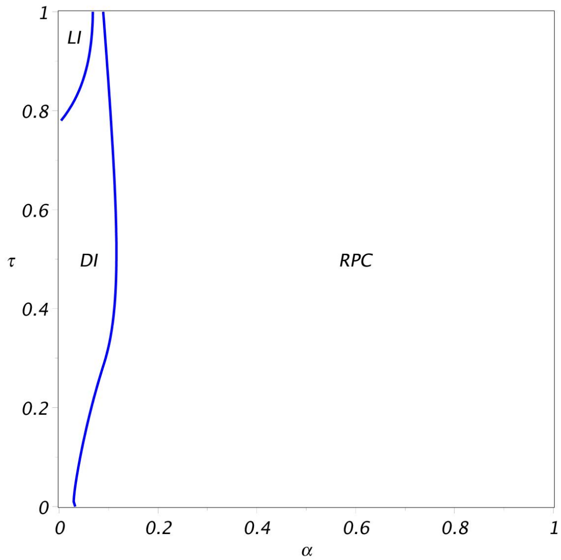
The arrangements under the RPC regime do not depend on the probability of runs α. Therefore, there will be no changes in computing V_n.

Comparison

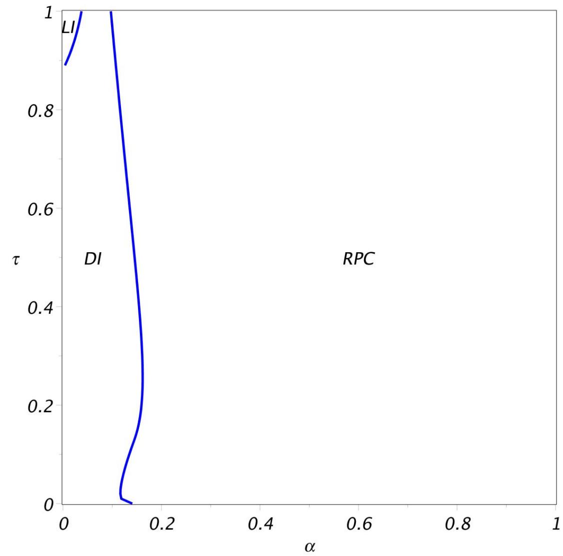
Similar to the main case in subsection 4.2, we assume the CRRA utility function $c^{1-\eta}/(1-\eta),\ \eta > 1$. The choice of κ is related to the issue of how large an unexpected shock to the DI fund may be. In 1989, the U.S. Congress instituted a target size for the DI fund: the ratio of the fund size to estimated insured deposits, called the Designated Reserve Ratio (DRR), had to be at least 1.25%. The Dodd-Frank Act increased that minimum ratio to 1.35%. Since 2011, FDIC has set a long-term goal for DRR to be 2%. An FDIC study revealed that if FDIC had followed the 2% target for DI fund, its balance would have not turned negative during the two episodes in the late 1980s-early 1990s and during the global financial crisis of 2007-2009. Since the pre-crisis ratio in both episodes was around 1.25%, we choose $\kappa = 2\%/1.25\% = 1.6$. We then compare the three regimes using the same parameter values as in Fig. 5. The results are presented in Fig. 7.

Two key differences are evident when comparing Fig. 5 and Fig. 7. First, we no longer observe the dominance of the LI regime for very high liquidation costs (τ) when the probability of bank runs (α) is relatively low. Instead, the LI regime shares this advantage with the DI regime. This can be attributed to the fact that when the LI fund is sufficient to cover all instances of bank troubles (as

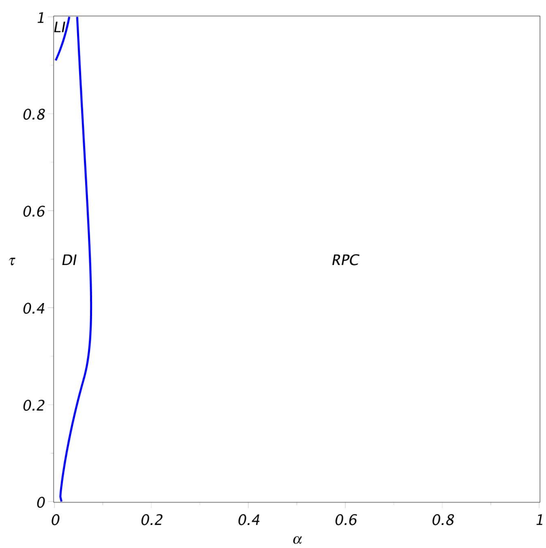
Figure 7: CRRA utility function $u(c) = \frac{c^{1-\eta}}{1-\eta}$; $\quad \tilde{\alpha} = \kappa\alpha = 1.6\,\alpha$



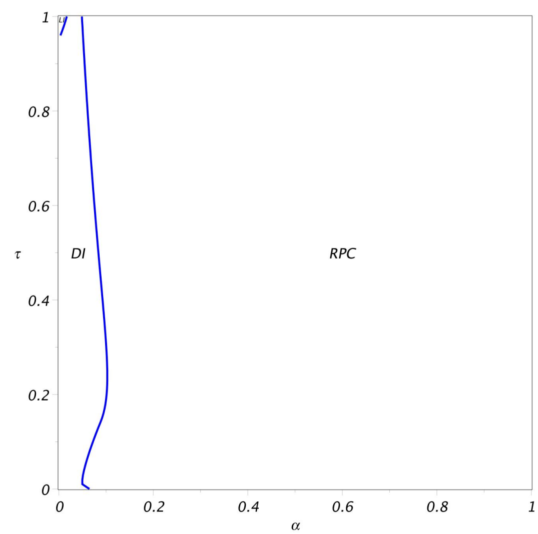
(a) $\pi = 0.5, \eta = 2.0, R = 1.5$



(b) $\pi = 0.5, \eta = 4.0, R = 1.5$



(c) $\pi = 0.5, \eta = 2.0, R = 1.2$



(d) $\pi = 0.5, \eta = 4.0, R = 1.2$

Each of the three areas represents dominance of a particular regime.

shown in Fig. 5), banks avoid liquidation costs, which are invariably present under the DI regime. However, when an unexpectedly large proportion of banks face difficulties, the LI fund becomes depleted, forcing unlucky troubled banks to liquidate all of their assets. Moreover, the remaining banks must liquidate a portion of their assets to pay taxes on behalf of their late depositors. For high liquidation costs (τ) and moderate probabilities of bank runs (α), these liquidation-associated costs appear excessive, resulting in the dominance of DI over LI. Second, Fig. 7 demonstrates that the insurance regimes (DI or LI) dominate the RPC regime over a narrower range of α compared to Fig. 5. This can be attributed to welfare losses caused by the unexpectedly large proportion of troubled banks (60% larger in Fig. 7). Nevertheless, both figures consistently show the dominance of the DI regime in the lower left corner and the LI regime in the upper left corner. It's worth noting that, as in Fig. 5, the combination $(\alpha, \tau) = (0.025, 0.28)$ falls within the DI dominance area in all four cases depicted in Fig. 7. These values of $\alpha = 0.025$ and $\tau = 0.28$ are consistent with the findings of Dávila and Goldstein (2023) and Granja et al. (2017), respectively.

5.2 On moral hazard

TBA

6 Conclusion

We have explicitly modeled ex-ante privately funded deposit insurance in a simple extension of the classic model in Diamond and Dybvig (1983). Two related private solutions to liquidity problem during a bank run are studied, liquidity insurance and runs preventing contracts. We find that when the probability of bank runs and liquidation costs are low, the DI scheme welfare dominates the other two regimes. In the case of CRRA utility function, we have shown that for realistic values of parameter values the DI regime is socially preferable among the three arrangements. We also find circumstances under which the other two regimes are dominant. There are several avenues along which this model can be extended further. We do not study moral hazard in this paper and intend to incorporate it in the future. It might also be of interest to explore other, more general incomplete privately funded deposit insurance schemes. In addition, one can study privately funded deposit insurance in a richer environment where banks fail for reasons other than bank runs.

Our study was, in part, motivated by the reluctance of the government to use taxpayers' money to rescue banks in March-April 2023, which, in turn, is driven by a significant public outcry against the bank bailout of 2008. There is no lack of private-market initiatives to tackle the vulnerability of the financial sector. The bank runs of 2023 in the U.S. indicate that the portions of deposits that were not insured are what led to the departure of funds from banks (i.e., those beyond the threshold of $250,000 cap). However, the private sector has devised a workaround to address this issue. Close to 3,000 regional banks participate in a network of "reciprocal deposit" swaps operated by IntraFi, which allows a bank to divide a large deposit into smaller pieces, under \$250,000 each, and deposit them into other participating banks (The Economist, April 15, 2023). If all large depositors used this service, the recent bank turmoil may have been avoided. Another example is the injection of \$30 billion into First Republic Bank by 11 largest U.S. financial institutions in March 2023 [27] (New York Times, March 16, 2023). However, a key difference between these examples and the two insurance schemes in this paper is that our deposit insurance and liquidity insurance programs are mandatory for banks. Our hope is that this study will contribute to the exploration of privately funded solutions to address the issue of financial fragility and provide a better understanding of their limitations.

Appendix

A.1 Proof of Proposition 1

Proof. Let us show that the area under the broken line CAB in Fig. 1 is the feasible set for the social planner's problem (3)–(6). It is sufficient to characterize the frontier of this set only. At the frontier, (5) always binds. And at least one of the two remaining constraints, (4) or (6) should bind. Consider these three cases.

- If we assume that all three constraints bind, then elimination of i_1 and i_2 leads to (16), which is line AB.

- Suppose (4) does not bind and (6) does. Then we can slightly increase i_2 so that (4) still does not bind but (6) becomes an inequality with the right-hand side being larger. Then we could increase c_1 without violating either inequality. But this violates our assumption that we are at the frontier of the feasible set.

- Suppose (6) does not bind and (4) does. Then it must be the case that $i_2 = 0$; otherwise we could could slightly decrease i_2 so that (4) becomes an inequality and (6) is still an inequality, in which case we could slightly increase c_1 without violating either constraint, which is impossible at the frontier. Then, if we impose $i_2 = 0$ and eliminate i_1 by using two binding constraints (4) and (5), we obtain (17), which is line CA.

Let us now prove (20). Recall the definition of δ in (19): $\delta = R(1-\alpha)/(\alpha\tau + 1 - \alpha)$. By Assumption 2, $\delta > 1$. Note that for a point (c_1, c_2) on line (16), if $c_1 = 1$ then $c_2 = \delta$. Thus, the bundle $(1, \delta)$ lies on segment AB in Fig. 1 since $1 > (1-\tau)/(1-\pi\tau)$.

[27] Although this effort was, in part, coordinated by the Treasury Secretary Janet Yellen.

We will argue that the optimal point lies to the right of point $(1, \delta)$ on AB. Indeed,[28]

$$\begin{aligned}
\delta \cdot u'(\delta) &= 1 \cdot u'(1) + \int_1^\delta \frac{\partial}{\partial c}\big[c \cdot u'(c)\big]\, dc \\
&= u'(1) + \int_1^\delta \big[u'(c) + c \cdot u''(c)\big]\, dc < u'(1).
\end{aligned}$$

The last inequality follows from $[u'(c) + c \cdot u''(c)] < 0$, which is a consequence of Assumption 1 and $u'(c) > 0$. We conclude that

$$\delta \cdot u'(\delta) < u'(1). \tag{A.1}$$

If expected utility (3) is indeed maximized on segment AB, i.e. the indifference curve is tangent to line (16), then the usual optimality condition yields

$$u'(c_1) = \frac{R(1-\alpha)}{\alpha\tau + 1 - \alpha}\, u'(c_2) = \delta \cdot u'(c_2)\,. \tag{A.2}$$

Note that the bundle $(1, \delta)$ does not satisfy (A.2) due to (A.1). Since $u'(c)$ is a decreasing function, and constraint trades off c_1 and c_2, the solution to the optimality condition (A.2) is when $c_1 > 1$ (and necessarily $c_2 < \delta$). Finally, $c_1 < c_2$ because $\delta > 1$. Thus, $1 < c_1 < c_2 < \delta$. ■

A.2 Derivation of constraint (22)

It is sufficient to characterize the frontier of the feasible set. There are two possibilities below.

(a) If all constraints (8)–(10) bind, then by eliminating i_1 and i_2 we obtain (22).

(b) Suppose at least one of the constraints (8)–(10) does not bind. Constraint (9) must bind: otherwise, since c_2 enters only this constraint among the three constraints, we could always increase c_2 without violating the other two, and this would increase utility level. Given that (9) binds, it is impossible for both (8) and (10) not to bind: we could then slightly increase c_1. Thus, there are only two possibilities: either (i) only (8) does not bind, or (ii) only (10) does not bind. We consider these possibilities separately.

   (i) Suppose only (8) does not bind. Then we can slightly increase i_2 so that (8) still does not bind; then (10) becomes a strict inequality. We then can slightly increase c_1, which contradicts the assumption that we are at the frontier of the feasible set.

   (ii) Suppose only (10) does not bind. Then we can slightly decrease i_2 so that (10) still does not bind; then (8) becomes a strict inequality. Since we then could slightly increase c_1, which would contradict the assumption of being at the frontier of the feasible set, we conclude that it must be the case that i_2 cannot be decreased, i.e. $i_2 = 0$. In such a case, (8) and (10) become

$$\begin{aligned}
\pi c_1 &= 1 - i_1\,, \\
c_1 &< 1 - i_1\,,
\end{aligned}$$

[28] The argument below to prove (A.1) borrows from Diamond and Dybvig (1983).

which is a contradiction since $\pi c_1 \leq \pi$.

A.3 Derivation of constraints (24) and (25)

Constraint (24) is obtained (as equality) when all three constraints (12)–(14) bind; the first constraint in (25) is obtained (as equality) when only (12) and (13) bind and $i_2 = 0$; the second constraint in (25) is obtained (as equality) when only (14) binds and $i_1 = 0$. It can be shown that these cases exhaust all possibilities at the frontier of the feasible set described by (12)-(14).

A.4 Proof of Proposition 2

Proof. Recall that we use V_d, V_l and V_n to denote the optimal values of expected utility in the DI, LI, and RPC problems respectively. Let $V_d(\alpha)$ and $V_l(\alpha)$ denote values of V_d and V_l associated with α; and note that V_n does not depend on α.

Step 1. Let us show that both $V_d(\alpha)$ and $V_l(\alpha)$ decrease in α. We will use the Envelope Theorem. Consider the DI problem (15)–(17). According to Proposition 1, only constraint (16) binds. Then the Lagrangian associated with this problem is

$$\begin{aligned}\mathcal{L}_d = &\big[\alpha + (1-\alpha)\pi\big] \cdot u(c_1) \;+\; (1-\alpha)(1-\pi) \cdot u(c_2) \\ &+ \lambda \left\{ \big[\alpha + (1-\alpha)\pi\big] c_1 + \frac{1-\pi}{R} \big[\alpha\tau + 1 - \alpha\big] c_2 \right\}.\end{aligned} \tag{A.3}$$

Let $x^d = (c_1^d, c_2^d, \lambda^d)$ be the critical point of $\mathcal{L}_d$ and (c_1^d, c_2^d) solves problem (15)–(17). According to the Envelope Theorem,

$$\frac{\partial V_d}{\partial \alpha} = \frac{\partial \mathcal{L}_d}{\partial \alpha}\Big|_{x=x^d}. \tag{A.4}$$

Then

$$\begin{aligned}\frac{\partial V_d}{\partial \alpha} = \frac{\partial \mathcal{L}_d}{\partial \alpha}\Big|_{x=x^d} &= (1-\pi) \cdot u(c_1^d) - (1-\pi) \cdot u(c_2^d) \\ &\quad + \lambda^d \left\{ -(1-\pi) \cdot c_1^d - \frac{1-\pi}{R}(\tau - 1) \cdot c_2^d \right\} \\ &= (1-\pi) \left\{ \big[u(c_1^d) - u(c_2^d)\big] + \lambda^d \left[-c_1^d + \frac{1-\tau}{R}\, c_2^d \right] \right\}.\end{aligned} \tag{A.5}$$

We want to demonstrate that the expression in (A.5) is negative. Since $1 - \pi > 0$, $\lambda^d > 0$ and $[u(c_1^d) - u(c_2^d)] < 0$ (because $c_1^d < c_2^d$), it suffices to show that $[-c_1^d + ((1-\tau)/R)\, c_2^d < 0$, which is equivalent to demonstrating

$$\frac{c_2^d}{c_1^d} < \frac{R}{1-\tau}. \tag{A.6}$$

Proposition 1 implies that

$$\frac{c_2^d}{c_1^d} < \frac{R(1-\alpha)}{\alpha\tau + 1 - \alpha}. \tag{A.7}$$

Since the right-hand side of (A.7) is smaller than that of (A.6), we have proven (A.6) and thus shown that $\partial V_d / \partial \alpha < 0$.

Similarly, the Lagrangian for the LI problem (21)–(22) is

$$\mathcal{L}_l = \pi \cdot u(c_1) \,+\, (1-\pi)\cdot u(c_2) \,+\, \lambda^l \left\{ 1 - \big[\alpha + (1-\alpha)\pi\big] c_1 - \frac{1-\pi}{R} c_2 \right\}, \tag{A.8}$$

and

$$\frac{\partial V_l}{\partial \alpha} = \frac{\partial \mathcal{L}_l}{\partial \alpha}\bigg|_{x=x^l} = -\lambda^l (1-\pi)\cdot c_1^l < 0\,.$$

Thus, indeed both $V_d(\alpha)$ and $V_l(\alpha)$ are decreasing in α.

Step 2. Let us show that $V_d(0) > V_n$ and $V_l(0) > V_n$. When $\alpha = 0$, both the DI problem and the LI problem become problem (1)–(2), the solution for which, as Diamond and Dybvig (1983) have shown, satisfies $1 < c_1 < c_2 < R$. In terms of Fig. 1, it means that the solution lies on segment AK and strictly to the right from point $(1, R)$. Since this is strictly outside of the feasible set for the RPC problem (which is the area below the $CADE$ broken line), we have $V_d(0) > V_n$ and $V_l(0) > V_n$.

Step 3. Let us show that $V_n > V_d(\alpha^*)$. We will do it in two stages.

(i) Let us show that $V_d(\alpha^*) = u(1)$. Proposition 1 states that $1 < c_1 < c_2 < \delta$, where δ as in (19). As $\alpha \to \alpha^*$ from below, point $(1, \delta)$ in Fig. 1 approaches point $D = (1, 1)$, i.e. $\delta \to 1$. Thus, at $\alpha = \alpha^*$ we have $c_1 = c_2 = 1$. Then indeed $V_d(\alpha^*) = u(1)$.

(ii) We know from Cooper and Ross (1998) that the solution to the RPC problem lies within AD. Let us show that point $D = (1, 1)$ is not optimal for the RPC problem. The marginal rate of substitution (MRS) is

$$MRS(c_1, c_2) = \frac{\pi\, u'(c_1)}{(1-\pi)\, u'(c_2)}\,,$$

and $MRS(1, 1) = \pi/(1-\pi)$. However, the slope of AD, as seen from eq. (24), is $(\pi\tau + R - 1)/[\tau(1-\pi)]$, which is greater than $\pi/(1-\pi)$.[29] Thus, the indifference curve at (1, 1) is flatter than the constraint, and thus the optimal point is to the left from (1, 1).

Since the value of V_n would be $u(1)$ if the optimal point were (1, 1), we conclude that $V_n > u(1) = V_d(\alpha^*)$.

The proof for $V_n > V_l(\alpha^*)$ is simpler. When $\alpha = \alpha^*$ and lines AB and AD coinside, segment CB, which is the constraint line for the LI problem, lies strictly below both segment AD and its continuation to the right. Since the solution to the RPC problem always lies either within segment AD or at point A, the utility level for the solution to the RPC problem must be higher than that for the LI problem; thus, $V_n > V_l(\alpha^*)$.

Step 4. Since $V_d(0) > V_n$, $V_d(\alpha^*) < V_n$, and $V(\alpha)$ is continuous and decreasing in α, there must exist $\hat{\alpha}_d \in (0, \alpha^*)$ such that $V_d(\alpha) > V_n$ for $\alpha < \hat{\alpha}_d$ and $V_d(\alpha) < V_n$ for $\alpha > \hat{\alpha}_d$. We can use the same argument for existence of $\hat{\alpha}_l \in (0, \alpha^*)$ in the case of V_l. ■

[29] We ignore the signs of all slopes throughout the paper, i.e. we deal with their absolute values.

A.5 Proof of Proposition 3

The following argument is used in both this proof and the proof of Proposition 4 below. For notational simplicity, let F^d denote the feasible set for the DI problem described by constraints (16)–(17). Consider these two utility functions:

$$\begin{aligned} U_d(c_1, c_2) &= \big[\alpha + (1-\alpha)\pi\big]\, u(c_1) + (1-\pi)(1-\alpha)\, u(c_2)\,, \\ U(c_1, c_2) &= \pi\, u(c_1) + (1-\pi)\, u(c_2)\,. \end{aligned}$$

The first one, U_d, is of course, the expected utility function in the DI problem (15)–(17). And the second, U, is the social welfare function that appears in both the LI problem (21)–(22) and the RPC problem (23)–(25). Consider the following two problems:

$$\max U_d \quad \text{s.t.} \quad (c_1, c_2) \in F^d\,, \tag{A.9}$$

$$\max U \quad \text{s.t.} \quad (c_1, c_2) \in F^d\,. \tag{A.10}$$

Let the optimal values of these utility functions in the two problems above be denoted by U_d^* and U^* respectively. Then we claim that for $\alpha > 0$,

$$U^* > U_d^*\,. \tag{A.11}$$

To prove it, let (c_1^d, c_2^d) denote the solution to problem (A.9). From Proposition 1 we know that $c_1^d < c_2^d$, and thus $u(c_1^d) < u(c_2^d)$. Then we have

$$U^* \geq U(c_1^d, c_2^d) > U_d(c_1^d, c_2^d) = U_d^*\,,$$

where the second inequality follows from the fact that U puts a greater weight on $u(c_2)$ than U_d does.

Now we turn to the proof of Proposition 3.

Proof. Refer to Fig. 2. As $\tau \to 1$, point A approaches point C. At $\tau = 1$, segment CA collapses to point C. Segment CB, which is segment AB at $\tau = 1$, goes above (1, 1) since $\delta = [R(1-\alpha)/(\alpha\tau + 1 - \alpha)] > 1$ by Assumption 2. Thus, segment $AB = CB$ lies above $AD = CD$ (see Fig. 3). Now we consider two separate comparisons.

(i) Since in the RPC problem and LI problem we maximize the same expected utility $\pi u(c_1) + (1-\pi)u(c_2)$, but the constraint for the LI problem (segment CB) lies strictly above that for the RPC problem (segment CD), we have $V_l > V_n$.

(ii) Note that the relevant constraints for the LI problem and DI problem coincide (segment AB). Then, we are dealing with the two problems above, (A.9) and (A.10). Using eq. (A.11), we conclude that $V_l > V_d$.

By continuity, both inequalities hold for τ that are near 1. ■

A.6 Proof of Proposition 4

Proof. Consider the RPC problem (23)–(24). We know that when $\tau = 0$, $(c_1^n, c_2^n) = (1, R)$ (see Cooper and Ross 1998). It is based on the fact that under Assumption 1, the first-best allocation is such that $1 < c_1 < c_2 < R$. Then using the curvature of indifference curves one can quickly conclude that the solution to the RPC problem when $\tau = 0$ is the corner solution $(1, R)$.

We now compare the slope S of the relevant constraint in the DI problem, segment AB in Fig. 1 described by eq. (16), with the marginal rate of substitution (or MRS) at point $(1, R)$ of the expected utility $U = \pi\, u(c_1) + (1-\pi)\, u(c_2)$ that is maximized in the RPC problem. It is easy to see that

$$MRS(1, R) = \frac{\pi u'(1)}{(1-\pi)u'(R)}\,, \tag{A.12}$$

$$S = \frac{[\alpha + (1-\alpha)\pi]R}{(1-\pi)[\alpha\tau + 1 - \alpha]}\,. \tag{A.13}$$

As α increases from 0 to α^* from (26), the slope S of segment AB increase from the slope of line CA to the slope of segment AD. The reason why (1, R) is the solution to the RPC problem (in fact, it is a corner solution) is that $MRS(1, R)$ is between these two extreme slope values. Thus there must exists $\alpha \in (0, \alpha^*)$ such that $MRS(1, R) = S$. (This situation is depicted in Fig. 4.) Let us denote such value of α by $\bar{\alpha}$ (to be found below).

For notational simplicity, let F^d denote the feasible set for the DI problem described by constraints (16)–(17) and represented by the area under the broken line CAB in Fig. 1.

For values of $\alpha > \bar{\alpha}$, maximization of the expected utility $U = \pi\, u(c_1) + (1-\pi)\, u(c_2)$ subject to F^d yields point $(1, R)$ as a solution because the indifference curve of U going through $(1, R)$ will always be above segment AB except point A. Now we refer to problems (A.9) and (A.10) and inequality (A.11). It is clear that

$$U^* = U(1, R) > U_d^* = V_d\,.$$

Since we established at the beginning that $(c_1^n, c_2^n) = (1, R)$, we have $V_n = U^* > V_d$. Thus, $V_n > V_d$ for $\alpha > \bar{\alpha}$.

Now note that when $\alpha > \bar{\alpha}$, segment CB, which is strictly below segment AB, will always be below the indifference curve of U that goes through point $(1, R)$ because segment AB is below that indifference curve. Since both the RPC problem and LI problem maximize the same expected utility, we conclude that $V_n > V_l$ for $\alpha > \bar{\alpha}$.

Let us now find $\bar{\alpha}$. It can be done by equating $MRS(1, R)$ and S from eq. (A.12) and (A.13) respectively and solving for α. This yields

$$\bar{\alpha} = \frac{\pi[u'(1) - Ru'(R)]}{(1-\pi)Ru'(R) + \pi u'(1)}\,,$$

which is eq. (27). ■

A.7 CRRA: Solving the three social planner's problems

We provide here solutions to the following three social planner's problems introduced in Section 2: the DI problem (15)–(16), the LI problem (21)–(22), and RPC problem (23)–(24). Note that the general problem below,

$$\max_{c_1,c_2}\left[a\cdot\frac{c_1^{1-\eta}}{1-\eta}+b\cdot\frac{c_2^{1-\eta}}{1-\eta}\right] \tag{A.14}$$

$$\text{s.t.}\quad m\cdot c_1+n\cdot c_2=d \tag{A.15}$$

has this solution:

$$c_1=\frac{d\cdot\theta}{m\cdot\theta+n},\quad c_2=\frac{d}{m\cdot\theta+n},\quad \text{where}\quad \theta=\left(\frac{n\cdot a}{m\cdot b}\right)^{1/\eta}. \tag{A.16}$$

Using this notation, the solutions to problems (15)–(16), (21)–(22), and (23)–(24), denoted by (c_1^d,c_2^d), (c_1^l,c_2^l), and (c_1^n,c_2^n) respectively, can be expressed as

$$c_1^i=\frac{\theta_i}{m_i\theta_i+n_i},\quad c_2^i=\frac{1}{m_i\theta_i+n_i},\quad i=d,l,n\,, \tag{A.17}$$

where

$$m_d=m_l=\alpha+(1-\alpha)\pi\,,\quad m_n=\frac{\pi\tau+R-1}{\tau+R-1}\,, \tag{A.18}$$

$$n_d=\frac{1-\pi}{R}(\alpha\tau+1-\alpha)\,,\quad n_l=\frac{1-\pi}{R}\,,\quad n_n=\frac{\tau(1-\pi)}{\tau+R-1}\,, \tag{A.19}$$

$$\theta_d=\left[\frac{\alpha\tau+1-\alpha}{(1-\alpha)R}\right]^{1/\eta},\quad \theta_l=\left[\frac{\pi}{[\alpha+(1-\alpha)\pi]R}\right]^{1/\eta},\quad \theta_n=\left[\frac{\tau\pi}{\tau\pi+R-1}\right]^{1/\eta}. \tag{A.20}$$

A.8 Proof of Proposition 5

Proof. Refer to the solutions to the three social planner's problems obtained in Appendix A.7. Note that $\theta_d<1$ by Assumption 2, and clearly, $\theta_l<1\,,\theta_n<1$. Therefore, in all three cases $i=d,l,n$, we have (i) $c_1^i<c_2^i$ since $c_1^i=\theta_i\cdot c_2^i\,,$; (ii) $c_1^i,c_2^i\to 1/(m_i+n_i)$ as $\eta\to\infty$ since $\theta_i\to 1$.

- Case V_d vs. V_l. Since $m_d=m_l$, and $n_d<n_l$ (recall that $\tau<1$), we have $c_1^l<c_2^l<c_1^d<c_2^d$ for large η. Since each V_i is a weighted average of $u(c_1^i)$ and $u(c_2^i)$, we can conclude that $V_l<V_d$.

- Case V_d vs. V_n. We have

$$m_n+n_n=\frac{\pi\tau+R-1}{\tau+R-1}+\frac{\tau(1-\pi)}{\tau+R-1}=1,$$

and thus $c_1^n,c_2^n\to 1/(m_n+n_n)=1$. Next,

$$m_d+n_d=(1-\alpha)\pi+\alpha+\frac{1-\pi}{R}(\alpha\tau+1-\alpha)<(1-\alpha)\pi+\alpha+(1-\pi)(1-\alpha)=1,$$

where the inequality follows from Assumption 2. Therefore, as $\eta\to\infty$, the consumption levels c_1^d,c_2^d converge to $1/(m_d+n_d)$, a number larger than 1. Thus, $c_1^n<c_2^n<c_1^d<c_2^d$ for large η. Since each V_i is a weighted average of $u(c_1^i)$ and $u(c_2^i)$, we can conclude that $V_n<V_d$.

■

A.9 Proof of Proposition 6

Proof. We will examine the behavior of the utility difference $\Delta V = V_d - V_l$. Treating V_i as functions of α, we want to establish $\Delta V(\alpha) > 0$ for small $\alpha > 0$. We know that $V_d = V_l$ when $\alpha = 0$ because, in this case, both problems maximize the same utility function subject to the same constraint. We first set $\tau = 0$. Let us show that

$$V_d'(0) > V_l'(0). \tag{A.21}$$

This will ascertain that $V_d(\alpha) > V_l(\alpha)$ for small $\alpha > 0$. By continuity, the same will hold for small $\tau > 0$.

Let us find $V_i'(\alpha)$. Using the framework of (A.14)–(A.15),

$$V_i(\alpha) = a_i \frac{(c_1^i)^{1-\eta}}{1-\eta} + b_i \frac{(c_2^i)^{1-\eta}}{1-\eta}, \tag{A.22}$$

where $a_d = (1-\alpha)\pi + \alpha,\ b_d = (1-\alpha)(1-\pi),\ a_l = \pi,\ b_l = 1-\pi$ and c_j^i are as in (A.17)–(A.20). Then differentiating $V_i(\alpha)$ and evaluating them at $\alpha = 0$ yields this:

$$V_d'(0) = -(1-\pi)\,\Omega^{\eta-1}\left[\frac{\Phi-1}{\eta-1} - -\frac{1}{\eta\,\Omega^2 R}\,Q\right], \tag{A.23}$$

$$V_l'(0) = -(1-\pi)\,\Omega^{\eta-1}\Phi, \tag{A.24}$$

$$Q = -\pi\,\Phi\,R^{-1/\eta}\,(\Phi\,\eta + \eta\,\tau - \eta - \tau) + \Phi\,(\eta\,\pi - \tau\,\pi - \eta) + \eta\,(1-\pi)\,(1-\tau), \tag{A.25}$$

$$\Omega = \frac{\pi R^{(\eta-1)/\eta} + 1 - \pi}{R}, \tag{A.26}$$

$$\Phi = R^{(\eta-1)/\eta}. \tag{A.27}$$

Therefore the derivative of the utility difference at $\alpha = 0$ is

$$\Delta V' \equiv V_d'(0) - V_l'(0) = (1-\pi)\Omega^{\eta-1}\left[-\frac{\Phi-1}{\eta-1} + \frac{1}{\eta\,\Omega^2 R}\,Q + \Phi\right].$$

We need to prove $\Delta V' > 0$, which, from the equation above, is equivalent to

$$\frac{(\eta-2)\Phi+1}{\eta-1} > -\frac{Q}{\eta\,\Omega^2 R}.$$

We first explore the case $\tau = 0$. Evaluating Q at $\tau = 0$ and using it in the inequality above and rearranging terms yields

$$R\,\frac{(\eta-2)\Phi+1}{(\eta-1)(\Phi-1)} > \frac{\pi\Phi R^{-1/\eta} + 1 - \pi}{\Omega^2}. \tag{A.28}$$

The right-hand side of (A.28) depends on π whereas the left does not. We want to find the maximum value of the right-hand side when π varies inside $[0,1]$ and make sure (A.28) holds with that value. Therefore we introduce function

$$f(\pi) = \frac{\pi\Phi R^{-1/\eta} + 1 - \pi}{\Omega^2}.$$

It can be shown that $f'(\pi) = A/\Omega^3$ where

$$A = -(\Phi R^{-1/\eta} - 1)\frac{\Phi-1}{R}\cdot\pi + \frac{1}{R}(\Phi R^{-1/\eta} - 2\Phi + 1).$$

Since $\Phi R^{-1/\eta} - 2\Phi + 1 < \Phi - 2\Phi + 1 = -\Phi + 1 < 0$, we conclude that $f'(0) < 0$. Since A is linear in π, it can change sign at most once. Thus, $\max_{\pi\in[0,1]} f(\pi)$ is attained at $\pi = 0$ or $\pi = 1$. Since

$f(0) = R^2 > R = f(1)$, we conclude that the maximum value of the right-hand side of (A.28) as a function of π is R^2. Inserting this value into (A.28) yields

$$\frac{(\eta-2)\Phi+1}{(\eta-1)(\Phi-1)} > R,$$

which becomes

$$(\eta-1)[\Phi-(\Phi-1)R]-(\Phi-1) > 0. \tag{A.29}$$

Since this should hold for any $\eta > 1$, we need to show that the function $F(\eta) = (\eta-1)[\Phi-(\Phi-1)R]-(\Phi-1) > 0$ for $\eta > 1$. Note that $F(1) = 0$, and thus it suffices to show that $F'(\eta) > 0$ for $\eta > 1$, i.e.

$$F'(\eta) = R\left\{R^{-1/\eta}\left[1-R-\ln R\cdot\frac{(\eta-1)(R-1)+1}{\eta^2}\right]+1\right\} > 0.$$

This is equivalent to showing that the function

$$G(\eta) = -R^{1/\eta}+R-1+\ln R\cdot\frac{(\eta-1)(R-1)+1}{\eta^2} < 0, \quad \eta > 1. \tag{A.30}$$

Let us find its derivative:

$$G'(\eta) = \frac{lnR}{\eta^3}\left[\eta\,(R^{1/\eta}-R+1)+2R-4\right].$$

It will be useful to introduce another function, $H(\eta)$, as follows:

$$H(\eta) = \eta\,(R^{1/\eta}-R+1)+2R-4. \tag{A.31}$$

It is clear that $G'(\eta) > 0$ iff $H(\eta) > 0$. Let us demonstrate the following properties of function G.

1. $G(1) < 0$. [Indeed, $G(1) = -1+\ln R < 0$ since $R \leq 2$.]

2. $\lim_{\eta\to\infty} G(\eta) = R-2 \leq 0$. [It is easy to establish.]

3. For large values of η, $G(\eta)$ is an increasing function.

   [Let us show it by arguing that $H(\eta) > 0$ for large η. Suppose first that $R < 2$. Denote $\rho = \lim_{\eta\to\infty}(R^{1/\eta}-R+1) = 2-R > 0$. Then the term $\eta(R^{1/\eta}-R+1) > \rho\eta$ grows at least linearly with respect to η and clearly $H(\eta) > 0$ for $\eta > (4-2R)/\rho$. Now suppose $R = 2$. Then

   $$\lim_{\eta\to\infty} H(\eta) = \lim_{\eta\to\infty}\frac{2^{1/\eta}-1}{1/\eta} = \lim_{x\to 0}\frac{2^x-1}{x} = \lim_{x\to 0}\frac{e^{\ln 2\cdot x}-1}{x} = \ln 2 > 0.$$

   Therefore, $H(\eta) > 0$ for large values of η.]

4. As $\eta \to \infty$, function $G(\eta)$ approaches its asymptotic value, which is nonpositive, from below, i.e. staying negative. [This follows from properties 2 and 3 above.]

The rest of the proof is by contradiction. Suppose that $G(\eta) > 0$ for some $\eta > 1$. This together with property 4 leads us to a conclusion that there are two values η_1 and η_2 with $1 < \eta_1 < \eta_2$ such that

(a) η_1 is a local maximum of G with $G(\eta_1) > 0$ and $G'(\eta_1) = 0$;

(b) η_2 is a local minimum of G with $G(\eta_2) < 0$ and $G'(\eta_2) = 0$.

Consider first η_1. From $G'(\eta_1) = 0$ it follows that $H(\eta_1) = 0$ and thus $R^{1/\eta_1} - R + 1 = (4 - 2R)/\eta_1$. Plug this into the expression for $G(\eta)$ in (A.30):

$$\begin{aligned} G(\eta_1) &= -R^{1/\eta_1} + R - 1 + \ln R \cdot \frac{(\eta_1 - 1)(R-1) + 1}{\eta_1^2} \\ &= \frac{2R-4}{\eta_1} + \ln R \cdot \frac{(\eta_1 - 1)(R-1)+1}{\eta_1^2} = \frac{1}{\eta_1}\left(2R - 4 + \ln R \cdot \frac{(\eta_1 - 1)(R-1)+1}{\eta_1}\right) > 0. \end{aligned}$$

Thus,

$$2R - 4 + \ln R \cdot \frac{(\eta_1 - 1)(R-1)+1}{\eta_1} > 0. \tag{A.32}$$

The function $g(\eta) = \frac{(\eta-1)(R-1)+1}{\eta}, \eta \geq 1$, is decreasing for $R < 2$; and $g(1) = 1$. Thus $\frac{(\eta_1-1)(R-1)+1}{\eta_1} < 1$. Plugging this into (A.32) yields

$$2R - 4 + \ln R > 0. \tag{A.33}$$

Now consider η_2. Since $G'(\eta_2) = 0$, we have $H(\eta_2) = 0$. Because η_2 is a local minimum, G' changes its sign from negative to positive; so does H. Therefore, H is increasing at $\eta = \eta_2$ and thus $H'(\eta_2) > 0$. Using (A.31), we find

$$H'(\eta) = R^{1/\eta} - R + 1 - R^{1/\eta} \cdot \frac{\ln R}{\eta}. \tag{A.34}$$

From $H(\eta_2) = 0$ and (A.31) we have $R^{1/\eta} - R + 1 = (4 - 2R)/\eta$. Plug this into (A.34) and use $H'(\eta_2) > 0$ to obtain $(4 - 2R)/\eta - R^{1/\eta} \ln R/\eta > 0$, which implies

$$4 - 2R - R^{1/\eta} \ln R > 0. \tag{A.35}$$

Note that function $h(\eta) = R^{1/\eta} \ln R$ is decreasing, and by taking the limit $\eta \to \infty$ we conclude that $R^{1/\eta} \ln R > \ln R$. Combining this with (A.35) we obtain

$$4 - 2R - \ln R > 0, \tag{A.36}$$

which contradicts (A.33). Thus, our assumption that $G(\eta) > 0$ for some $\eta > 1$ is incorrect; therefore $G(\eta) < 0$ for all $\eta > 1$. This means that $F'(\eta) > 0$ and $F(\eta) > 0$, which finally leads to $\Delta V'(\eta) > 0,\ \eta > 1$. This is a result for $\alpha = 0$. By continuity, it also holds for small positive values of α as well. Since this conclusion is true for $\tau = 0$, by continuity it is correct for small positive values of τ too. ∎

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