Explaining the Compensation of Superstar CEOs*

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Abstract

While agency theory had an enormous influence on managerial incentives, most observed compensation plans do not make the economic primitives of the underlying agency problem explicit. In this study, we derive closed-form expressions for the cost of effort, the manager's surplus, and the contribution of effort to firm value, as a function of observable characteristics of the contract. We develop simple interpretable expressions in the risk-neutral limit, and show that optimality can be rationalized under a condition satisfied by most realistic contracts. We apply the model to the case Tornetta v. Musk, which voided $54 billion of incentive pay, and show that, according to the parameters of the contract, Musk (i) did not have significant bargaining power, (ii) was primarily compensated for large opportunity costs, (iii) was not expected to contribute at levels close to ex-post performance, and (iv) inducing effort would have been infeasible or prohibitively expensive without options. The model offers a tractable framework to understand the assumptions underlying large high-powered incentive plans for "superstar" CEOs.

Keywords: CEO, compensation, incentives, agency, option.

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1 Introduction

Superstar CEOs are executives who have achieved a high level of prestige in the public eye. Their success or failure is seen as closely tied to the fate of the company they manage. These managers typically receive significantly higher pay and incentives than their peers and benefit from potentially massive bargaining power due to their control of the board. This can lead to decisions that prioritize their interests over those of the shareholders rather than maximizing the company's value (Malmendier and Tate 2009). Understanding their compensationis difficult, as we cannot observe how they would perform under different incentive plans, and the rationale behind their pay is not self-evident.

In this study, we develop a methodology to infer the primitives of the agency problem facing the board, using observed compensation and the distribution of performance measures. We allow for the possibility that superstar CEOs (i) have low risk-aversion, (ii) have expressed their intention to stay even if the incentive contract was rejected, and (iii) own a nontrivial share of the equity.

In our context, these assumptions are reasonable, considering that these managers are typically very affluent and possess a substantial portion of unrestricted shares, which they would have likely sold had their aversion to risk been considerable; further, they likely receive substantial personal benefits of control. Existing research has shown that these primitives can be inverted under constant absolute risk-aversion preferences (Margiotta and Miller 2000; Gayle and Miller 2009; Gayle and Miller 2015; Gayle, Li, and Miller 2022); following these earlier studies, we obtain simple interpretable expressions for the cost of effort, bargaining power, and the contribution of effort to performance metrics for general utility functions and in the risk-neutral limit. Further, we extend the analysis to continuous effort and possibly unbounded compensation structures and develop a straightforward method for parametric identification such that, as a possible application, effort changes the drift and volatility of a geometric price process.

We also provide several identification results. First, we prove that near-risk neutrality, which implies that the agency problem's cost is small, is difficult to falsify for most observed compensation structures. Under constant relative risk-aversion, near risk neutrality is compatible with any contract in which the principal and manager receive monotonic payoffs or, under constant absolute risk-aversion, as long as the variability in total pay tends to be at least one order of magnitude lower than the variability of firm value. Second, we show that a continuous effort model is required to explain the potentially unbounded compensation structures that we observed in practice, that the first-order approach is appropriate in these settings, and that the model identifies the marginal contribution of effort to firm value per dollar of effort cost.

Debates about appropriate levels of incentives have been ongoing since at least since Jensen and Murphy 1990, who observed commonly-seen incentive levels at 0.1% of shareholder value creation, at odd with performance pay in other economic activities (e.g., entrepreneurship, sales, single-proprietorships, etc.), may not be sufficient to align managerial incentives. Although incentives have increased dramatically since their study, the question is largely unresolved in part due to the absence of methods to infer the assumptions embedded in compensation arrangements and given that the specification of the incentive model can justify a range of incentive levels (Haubrich 1994; Hall and Liebman 1998). Therefore, an approach that uses information from observed contracts and the economic structure from an explicit incentive design is needed. We propose to bridge this gap in the literature and provide a simple methodology that would allow researchers and practitioners to understand what quantitative assumptions are implied by a contract. The closed-form nature of the mapping allows any contract to be inverted into intuitive economic primitives, such as the surplus realized by the manager, the value of effort, and the monetary cost of effort.

We apply the model to an example that features incentives that are orders of magnitude higher than these levels, namely the compensation of Elon Musk as CEO of Tesla. Another aspect of this example is that a recent Delaware court case decision, Tornetta v. Musk, annulled the $54.8 billion incentive payout. The court found that the defense could not explain its hypotheses and prove that the 2018 incentive contract was in the firm's best interest. This case illustrates the objective of the methodology, as the defense did not offer quantitative measurements as to what was implied in the contract about Musk's value to Tesla, his bargaining power, and the necessary opportunity cost of aligning incentives given Musk's other interests. This setting is also ideal for the model's assumptions because Musk has shown little risk-aversion through his entrepreneurial endeavors and retention of substantial unrestricted equity in Tesla (21.9%) and received a total compensation measured by Tesla at $2.615 billion, or many times that of closest peers. However, while we use Tesla as an example, the methodology can be applied to other superstar CEOs sharing similar characteristics.

Consistent with our theoretical analysis, the model rationalizes the compensation near risk neutrality, but the inferred primitives reveal somewhat surprising novel insights. The expected contribution of Musk's effort to value is high, at around $30B, yet it is an order of magnitude less than realized performance. We also find that the contract is designed consistent with Musk bearing a substantial cost to maximizing shareholder value relative to pursuing other interests, at about $6.6B, which implies that the surplus of Musk from participating in the contract is *negative*, at $-6.6B, relative to being able to sell all shares at the current market price. We interpret this result as implying that, according to the assumptions embedded in the contract, Musk did not have bargaining power, and, in fact, his bargaining power was lowered by the potential loss from his existing share ownership should he announce he was leaving Tesla. This result suggests that sizeable prior share ownership may reduce bargaining power.

Superstar contracts are highly complex. They are defined over multiple performance measures and have sophisticated performance vesting conditions. Some of this complexity—it has been argued—is aimed at obfuscating the total value of the CEO's compensation from shareholders and other market participants.

We analyze the role of contract complexity in maximizing firm value. In particular, we consider the cost to the firm from implementing simple restricted-stock plans with time-vesting structures, instead of the actual optimal contract. We ask how much the firm loses from restricting attention to simple contracts and whether effort can be implemented in the absence of exotic options and performance vesting. Our preliminary findings suggest that options are indispensable to implement effort: simple contracts don't have enough incentive power to implement effort.

We also consider the relevance of (accounting) operational milestones and their role. We show that accounting milestones have a small effect on the expected value of CEO compensation, as they do not significantly affect the probability of vesting in equilibrium. If anything, accounting milestones relax incentive compatibility constraints by making it less attractive for the manager to deviate and choose low effort.

2 The Model

2.1 General Preferences

A risk-averse manager has a utility function $u(w;e)$, where $u(w;e)$ is twice-differentiable, increasing and concave in w, has a constant sign, satisfies Inada conditions, and depends on end-of-period wealth w and an unobservable effort action $e \in \{0,1\}$. Conditional on e, the firm achieves a contractible performance measure $\mathbf{x} \in X \subseteq \mathbb{R}^n$, with cumulative probability (resp., density) function $F(.|e)$ (resp., $f(.|e)$). There exists a performance level $\overline{\mathbf{x}}$ such that $\lim_{x\to\overline{x}} f(\mathbf{x}|0)/f(\mathbf{x}|1) = 0$, that is, a performance sufficiently close to $\overline{\mathbf{x}}$ indicates high effort.

Suppose that $u(w;1) < u(w;0)$ for any w, so that the manager prefers low effort. In this model, $e = 0$ can be equivalently interpreted as private benefits from engaging in pet projects or, more generally, maximizing objectives other than those valuable to the firm owners. Assume for now that the cost of effort is multiplicative and, with a slight abuse in language, we write $u(w;1) \equiv Au(w)$ and $u(w;0) \equiv u(w)$. The multiplicative formulation nests, as special cases, models with constant absolute risk-aversion (or exponential utility) with "in-the-utility" cost of effort (Margiotta and Miller 2000; Gayle and Miller 2009; Feltham and Xie 1994).

The firm is risk-neutral and sets a wage $w(.)$ to maximize shareholder value

$$v(w, \mathbf{x}) = \phi(\mathbf{x}) - w(\mathbf{x}),$$

where $\phi(.)$ is a known continuous function (for example, a market price). This maximization is subject to two constraints. First, given that we aim to explain observed non-trivial compensation schedules, we assume that the firm prefers to elicit high effort $e = 1$. The resulting incentive constraint can then be written as an inequality such that the manager achieves a higher utility when choosing effort:

$$\int Au(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} \geq \int u(w(\mathbf{x}))f(\mathbf{x}|0)d\mathbf{x}. \tag{IC}$$

Second, the bargaining game between the firm and the manager is determined by the manager's outside opportunities and how the gains from the employment relationship are shared. As a result of this bargaining game, the manager achieves a surplus $u(\underline{w})$ (Göx and Hemmer 2020). By definition, this implies that the contract satisfies an individual rationality constraint:

$$\int Au(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} \geq u(\underline{w}). \tag{IR}$$

If the firm had all bargaining power (defined as making a single take-it-or-leave-it offer), one could interpret $\underline{w}$ as the certainty equivalent of the best alternative occupation in a labor market with competition across differentiated managers (Gabaix and Landier 2008); however, $\underline{w}$ may also capture a share of the total surplus of the match, consistent with matching models (Mortensen and Pissarides 1994). Since the alternative occupation cannot be observed empirically, we remain agnostic about the interpretation of $\underline{w}$ and view it as a cost of employing the manager.[1]

The constraint (IC) must bind given any contract that is not constant since otherwise, it would violate the Arrow-Borsch risk-sharing conditions. Without loss of generality, the

[1]Under the managerial power hypothesis, managers with bargaining power may obtain more than their outside option and, in certain circumstances, set their pay subject to a minimum return for shareholders (Bertrand and Mullainathan 2001; Göx and Hemmer 2020). Note that the managerial power hypothesis is nested within this formulation as long as $\underline{w}$ is defined as the surplus achieved by the manager. Formally, the problem formulation as the principal maximization subject to the manager's participation is the dual of the manager maximization subject to the principal's participation (Mas-Colell, Whinston, and Green 1995).

constraint (IR) can also be set to bind as long as $\underline{w}$ is defined as the equilibrium utility of the manager in the optimal contract. In what follows, to interpret the cost of effort in monetary units (rather than in utils), we define

$$\mathcal{C} \equiv u^{-1}(u(\underline{w})/A) - \underline{w}, \tag{1}$$

such that $\mathcal{C}$ represents a compensating differential for the manager to exert effort relative to no effort. Specifically, this differential satisfies $Au(\underline{w} + \mathcal{C}) = u(\underline{w})$ and is the amount to be paid to elicit effort in first-best (if the effort were contractible), assuming a manager base utility of $u(\underline{w})$.

Our objective will be to recover the parameters of the model $(\underline{w}, A, \mathcal{C})$ from the observed wage schedule $w(.)$ and density of the performance measure $\mathbf{x}$ conditional on high effort. As shown by Gayle and Miller 2015, the risk-aversion is usually not point identified, so we shall assume it is known; however, later on, we will consider a baseline in which risk-aversion is very low, consistent with the postulate that superstar managers have high levels of risk tolerance. The following Proposition offers a simple closed-form expression of the agency parameters for any observed contract.

Proposition 1 *For a given wage $w(.)$ and distribution of contractible information $f(.|1)$, the counter-factual distribution of performance with $e = 0$ is given by*

$$f(\mathbf{x}|0) = f(\mathbf{x}|1)\frac{\gamma - \frac{1}{u'(w(\mathbf{x}))}}{\gamma - \alpha}, \tag{2}$$

where $\alpha \equiv \mathbb{E}(1/u'(w(\mathbf{x})))$ and $\gamma \equiv 1/u'(w(\overline{\mathbf{x}}))$. Then, the manager's surplus is given by $\underline{w} = u^{-1}(\mathbb{E}(u(w(\mathbf{x}))|e = 0))$ and the cost of effort is $A = \mathbb{E}(u(w(\mathbf{x}))|e = 0)/\mathbb{E}(u(w(\mathbf{x}))|e = 1)$, that is, in monetary terms,

$$\mathcal{C} = u^{-1}(\mathbb{E}(u(w(\mathbf{x}))|e = 1)) - u^{-1}(\mathbb{E}(u(w(\mathbf{x}))|e = 0)). \tag{3}$$

For a given wage profile $w(\mathbf{x})$ and density of the observed performance measure $f(\mathbf{x}|1)$, equation (2) offers a closed-form expression for the counter-factual distribution of effort; then, the expressions for the manager surplus and cost of effort follow immediately from (IC) and (IR). In the right-hand side of (2), the shape of the compensation determines the implied probability of each outcome conditional on no effort. Inside the expression for the likelihood ratio $\theta(w(\mathbf{x})) \equiv f(\mathbf{x}|0)/f(\mathbf{x}|1)$, a relatively low (high) compensation implies that the outcome is relatively more (less) likely under no effort. One can further characterize the effect of the marginal change in observed pay on the likelihood ratio as

$$\theta'(w(\mathbf{x})) = -\frac{1}{u'(w(\mathbf{x}))(\gamma - \alpha)} \frac{-u''(w(\mathbf{x}))}{u'(w(\mathbf{x}))}. \tag{4}$$

The right-most term is the manager's coefficient of absolute risk-aversion and implies that greater concavity in the utility function tends to magnify the effect of a higher wage on the likelihood ratio; intuitively, risk-aversion penalizes the principal for offering more risky compensation by requiring a risk premium and, therefore, must be offset with more information about effort. Finally, we also measure the contribution of managerial effort to the output as defined by the increase in output due to effort:

$$\Delta \equiv \mathbb{E}(\phi(\mathbf{x})|e=1) - \mathbb{E}(\phi(\mathbf{x})|e=0) = \frac{cov(\phi(\mathbf{x}), \frac{1}{u'(w(\mathbf{x}))})}{\gamma - \alpha}. \tag{5}$$

Δ can be written in terms of the covariance of the manager's inverse marginal utility (a convex increasing function) and the firm's objective.

2.2 Interpreting Observed Compensation

Among risk preferences, constant relative risk-aversion preferences have been observed to capture risk-taking behavior and exhibit the (plausible) property that risk tolerance increases in wealth. Using evidence from early options exercises, Brenner 2015 finds that the median executive in a sample of large publicly traded firms has close to a logarithmic risk preference (with relative risk-aversion of 0.911) but also documents substantial heterogeneity with risk-aversions ranging from 0.11 at the 10^{th} percentile versus 6.17 at the 90^{th} percentile. He also finds that about 9% of all executives exercise their vested options as if near risk-neutral, which we will revisit in the next section when considering low risk-aversion.

In this section, we develop additional interpretations assuming a power utility function of the form $u(w) = w^{1-r}/(1-r)$, where r indicates the coefficient of relative risk aversion. Then, the distribution of output without effort is

$$f(\mathbf{x}|0) = f(\mathbf{x}|1) \frac{1 - (\frac{w(\mathbf{x})}{w(\overline{\mathbf{x}})})^r}{1 - \mathbb{E}((\frac{w(\mathbf{x})}{w(\overline{\mathbf{x}})})^r)} \tag{6}$$

and depends only the ratio of forfeited compensation $\frac{w(\mathbf{x})}{w(\overline{\mathbf{x}})}$. The manager's surplus and monetary cost of effort are then given by a fraction of the maximal wage $\underline{w} = \lambda_0 w(\overline{\mathbf{x}})$ and

$\mathcal{C} = (\lambda_1 - \lambda_0)w(\overline{\mathbf{x}})$ with

$$\lambda_0 \equiv \mathbb{E}((\frac{w(\mathbf{x})}{w(\overline{\mathbf{x}})})^{1-r}\frac{1-(\frac{w(\mathbf{x})}{w(\overline{\mathbf{x}})})^r}{1-\mathbb{E}((\frac{w(\mathbf{x})}{w(\overline{\mathbf{x}})})^r)})^{1/(1-r)}, \quad \lambda_1 \equiv \mathbb{E}((\frac{w(\mathbf{x})}{w(\overline{\mathbf{x}})})^{1-r})^{1/(1-r)}. \tag{7}$$

These properties yield two somewhat surprising interpretations when observing differences in contract arrangements. First, suppose that two firms feature a similar distribution of performance measures conditional on effort $f(\mathbf{x}|1)$ and assume that their managers have similar risk preferences (e.g., have similar characteristics or backgrounds); however, one firm chooses a contract $w(\mathbf{x})$ while the other firm chooses a contract $w_2(\mathbf{x}) = \rho w(\mathbf{x})$ with greater performance sensitivity $\rho > 1$. In linear-normal models with constant absolute risk-aversion, this is usually interpreted as a more precise performance measure (Banker and Datar 1989; Feltham and Xie 1994). By contrast, in (6), the likelihood ratio does not depend on ρ, implying that the performance sensitivity, holding the convexity of the contract fixed, does not contain *any* information about the quality of performance measures or the contribution of effort to output in (5).

Second, existing research documents a weak link between performance pay and the variability of performance measures (Prendergast 2000). To examine this question, suppose that two firms issue the same compensation $w(\mathbf{x}) = w_2(\mathbf{x})$ but the second firm features a performance measure $\mathbf{x}_2 = \rho\mathbf{x}$, with $\rho > 1$ indicating more variability relative to the performance performance measure $\mathbf{x}$. From (6), the two firms feature the same likelihood ratios $f_2(\mathbf{x}|1)/f_2(\mathbf{x}|0) = f(\mathbf{x}|1)/f(\mathbf{x}|0)$ and, hence, the interpretation of this choice is that the performance measures contain the same information about effort. As to the contribution of effort,

$$\Delta = \int (1 - \frac{f(\mathbf{x}|0)}{f(\mathbf{x}|1)})\phi(\mathbf{x})f(\mathbf{x}|1)d\mathbf{x} = \int (1 - \frac{f_2(\mathbf{x}_2|0)}{f_2(\mathbf{x}_2|1)})\phi(\mathbf{x}_2/\rho)f_2(\mathbf{x}_2|1)d\mathbf{x}_2 < \Delta_2, \tag{8}$$

implying that the firm with more variability in its performance measure must have a greater contribution of effort to performance.

Logarithmic utilities, in addition to matching Brenner's median estimate, have a long history in terms of approximations of behavior under uncertainty and given their ease of interpretation (Kelly 1956; Rubinstein 1976). Under this assumption, the likelihood ratio is obtained as the forfeited compensation (defined as maximum pay minus the actual compensation) relative to the average forfeited compensation:

$$f(\mathbf{x}|0) = f(\mathbf{x}|1)\frac{w(\overline{\mathbf{x}}) - w(\mathbf{x})}{w(\overline{\mathbf{x}}) - \mathbb{E}(w(\mathbf{x}))}, \tag{9}$$

which implies the stronger prediction that any sensitivity or an increase in fixed pay, that is $w_2(\mathbf{x}) = \rho_0 + \rho_1 w(\mathbf{x})$ imply the same likelihood ratio and, hence, the same contribution of effort Δ. The contribution of manager effort further simplifies to

$$\Delta = \frac{cov(w(\mathbf{x}), \phi(\mathbf{x}))}{w(\overline{\mathbf{x}}) - \mathbb{E}(w(\mathbf{x}))} \tag{10}$$

increasing in the ratio of the performance pay $cov(w(\mathbf{x}), \phi(\mathbf{x}))$ to the average forfeited pay. The counterfactual distribution depends only on the convexity of the wage rather than changes in levels or slope.

2.3 Risk-Neutral Limits under HARA preferences

We are interested next in parameter estimates when the manager is sufficiently close to risk-neutrality, since this may be descriptive of managers receiving extreme levels of risk in their compensation.[2] Consider the following formulation of HARA preferences, which include most common utility functions such as constant absolute risk-aversion (when $b = 1$ and $r \to -\infty$), constant relative risk-aversion ($b = 0$), or quadratic utility functions ($a < 0 < b$ and $r = 2$):

$$u(w) = \frac{1-r}{r}\left(\frac{aw}{1-r} + b\right)^r, \tag{11}$$

where w takes values over the domain and $\frac{aw}{1-r} + b \geq 0$. For now, we consider the sole limit on one parameter in (11) that converges to risk-neutrality, which must involve $r \to 1$. For now, we are delaying the limit under constant risk-aversion since it involves evaluating two limits sequentially. Hereafter, denote $W \equiv \mathbb{E}(w(\mathbf{x}))$ as the expected wage.

Proposition 2 *For any preference of the form (11), the cost of effort and the counter-factual distribution of output satisfy*

$$\lim_{r\to1} A = A_0^H \equiv \frac{\int \frac{w(\mathbf{x})}{W} \ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x})}) f(\mathbf{x}|1) d\mathbf{x}}{\int \ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x})}) f(\mathbf{x}|1) dx}, \quad \lim_{r\to1} f(\mathbf{x}|0) = f(\mathbf{x}|1)\frac{\ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x})})}{\int \ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x}')}) f(\mathbf{x}'|1) d\mathbf{x}'}, \tag{12}$$

[2] Note that we mean here a manager whose preferences can be approximated by low risk aversion, not a risk-neutral manager. Risk-neutrality implies that there are many optimal contracts: any contract satisfying (IR) at equality and (IC) as an inequality will be optimal. However, many of these contracts are not continuous limits of contracts with low risk-aversion and would require a knife edge such that the preference is precisely linear.

and the monetary cost of effort and manager surplus converge to a fraction of the expected wage: $\mathcal{C} \to W(1 - A_0^H)$ *and* $\underline{w} \to WA_0^H$.

Under HARA preferences, the cost of effort can be written as a weighted probability of each wage payment relative to the average wage $\frac{w(\mathbf{x})}{W}$. The weights $\ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x})})$ are similar to a stochastic discount factor in asset pricing, except that they no longer capture a reduction in the probability of (favorable) states due to risk-aversion but an implicit discount implied by the lower payments given (unfavorable) events.

The same logic applies under constant absolute risk-aversion. Still, one has to be careful in the limiting argument given that this type of preference is obtained by taking $r \to \infty$ and is therefore incompatible with an approximation of risk-neutrality with r. Instead, to converge to risk-neutrality holding absolute risk-aversion constant, one needs to first write $u(w) = -e^{-aw}$, by setting $b = 1$ and $r \to -\infty$, and then take the limit of (2) as $a \to 0$. Under CARA preferences, the model features a cost of effort "in utility" in the sense that the monetary cost of effort simplifies to

$$\begin{aligned}\mathcal{C} &= u^{-1}(\mathbb{E}(u(w(\mathbf{x}))|e=1)) - u^{-1}(\mathbb{E}(u(w(\mathbf{x}))|e=0)) \\ &= u^{-1}(\mathbb{E}(u(w(\mathbf{x}))|e=1)) - u^{-1}(\mathbb{E}(u(w(\mathbf{x}) + \ln(A)/a)|e=1)) = \ln(A)/a,\end{aligned}$$

and only depends on the inferred coefficient A and the managerial absolute risk-aversion a, which indicates that the manager experiences a disutility equivalent to a wage reduction by $\ln A/a$ independently of any wealth level.[3]

Proposition 3 *Under constant absolute risk-aversion, i.e.,* $u(w) = -e^{-aw}$*, the cost of effort, the counter-factual distribution of output, and the monetary cost of effort near risk-neutrality are given, respectively, by*

[3] As shown in Fellingham, Newman, and Suh 1985 and Margiotta and Miller 2000, under exponential preferences, any optimal contract with commitment is equivalent to a sequence of short-term contracts (as we use here). While this equivalence does not generally hold for other preferences, Fudenberg, Holmstrom, and Milgrom 1990 show that if the manager has free access to capital markets to smooth intertemporal consumption, the optimal long-term contract with commitment can be similarly represented as a sequence of short-term contracts; however, an important difference with exponential preferences, is that the principal may not have a linear utility, especially if higher pay makes effort more costly to elicit in future periods. If the principal has a utility function $V(\phi(\mathbf{x}) - w(\mathbf{x}))$, all terms in (2) are

$$f(\mathbf{x}|0) = f(\mathbf{x}|1)\frac{\gamma - \frac{V'(\phi(\mathbf{x}) - w(\mathbf{x}))}{u'(w(\mathbf{x}))}}{\gamma - \alpha}, \tag{13}$$

where $\alpha \equiv \mathbb{E}(V'(\phi(\mathbf{x}) - w(\mathbf{x}))/u'(w(\mathbf{x})))$ and $\gamma \equiv V'(\phi(\overline{\mathbf{x}}) - w(\overline{\mathbf{x}}))/u'(w(\overline{\mathbf{x}}))$, where (typically) the principal's utility is concave (Spear and Srivastava 1987). Unfortunately, this formulation introduces an extra parameter to recover $V(.)$, which is not well-identified. In the risk-neutral limit discussed later, it is readily shown that the principal's preference in this equivalent contract must become linear.

$$\lim_{a\to 0} A = 1, \quad \lim_{a\to 0} f(\mathbf{x}|0) = f(\mathbf{x}|1)\frac{w(\overline{\mathbf{x}}) - w(\mathbf{x})}{w(\overline{\mathbf{x}}) - W}, \quad \lim_{a\to 0} \mathcal{C} = \frac{Var(w(x))}{w(\overline{\mathbf{x}}) - W}, \tag{14}$$

and $\lim_{a\to 0} \underline{w} = W - \lim_{a\to 0} \mathcal{C}$.

In the particular case of constant absolute risk-aversion, the counterfactual distribution in the limit is identical (and hence, empirically indistinguishable) to the model with logarithmic utilities in equation (9). The cost of effort in utils must become small as the manager becomes more risk tolerant ($\lim_{a\to 0} A = 1$), which might suggest that the cost of the effort is small from the firm's perspective. However, this is not the case because the utility function flattens near risk-neutrality, i.e., $u(w) \to -1$, so the extra compensation required for each unit of disutility of effort becomes unbounded. In the limit, the two effects offset so that the monetary cost of effort converges. The resulting expression reveals the role of convexity in the compensation and features a ratio of the variance of the compensation scaled by the average forfeited pay.

It may seem paradoxical that there is a well-defined risk-neutral limit, noting that if we evaluate the manager's preference near risk-neutrality $u'(w) \to 1$, the first-order condition on the principal's problem Lagrangian has a form:

$$\frac{1}{u'(w(\mathbf{x}))} = A(\lambda + \mu) - \mu \frac{f(\mathbf{x}|0)}{f(\mathbf{x}|1)}, \tag{15}$$

which might suggest that for a multiplier $\mu > 0$, the contract puts arbitrarily large pay on outcomes with the highest likelihood ratios, similar to live-or-die contracts (Innes 1990). However, this logic is incorrect because the incentive multiplier $\mu \to 0$ must vanish as the manager becomes risk-neutral, as there is no longer any risk premium paid for inducing effort: equation (15) becomes $1 = \lambda$ and becomes unhelpful in characterizing the optimal contract. In fact, given the likelihood ratio in Proposition 2, the contract $w(\mathbf{x})$ verifies (by construction) both (IC) and (IR) at equality and yields a residual surplus to the principal:

$$\mathbb{E}(\phi(\mathbf{x}) - w(\mathbf{x})) = \mathbb{E}(\phi(\mathbf{x})) - \underline{w} - \mathcal{C}, \tag{16}$$

identical to the surplus with observed effort and, therefore, must be an optimal contract. More generally, when risk-neutrality is attained, any contract that binds (IR) and satisfies (IC) is optimal. In other words, the risk-neutral limit characterized in Proposition 2 selects the (unique) limit contract that can be derived as the limit of contracts with vanishing risk-aversion. It can also be interpreted as a risk-neutral contract robust to a small amount of risk aversion.

An interesting Corollary to Proposition 2 is that contract optimality cannot be easily ruled out for sufficiently low risk-aversion. Gayle and Miller 2015 note that the observation of a contract that is not constant implies, by revealed preferences, an inequality constraint of the form:

$$\int (\phi(\mathbf{x}) - w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} \geq \int \phi(\mathbf{x})f(\mathbf{x}|0)d\mathbf{x} - \underline{w}. \tag{17}$$

Proposition 4 *Under the risk-neutral limit with $r \to 1$, (17) is satisfied if and only if $cov(\phi(\mathbf{x}) - w(\mathbf{x}), \ln(w(\mathbf{x}))) > 0$. Under the risk-neutral limit of absolute risk-aversion preferences, (17) is satisfied if and only if $\frac{Var(w(x))}{w(\overline{\mathbf{x}})-W} \leq cov(\phi(\mathbf{x}) - w(\mathbf{x}), w(\mathbf{x}))$.*

Proposition 4 demonstrates that if the residual claim to the firm and to the manager are increasing in performance, so that both parties always benefit from greater $\phi(\mathbf{x})$, there exists a preference rationalizing the contract close to risk-neutrality. Further, this solution can feature arbitrarily small agency costs since transferring risk to the manager does not command a risk premium. When restricting preferences to constant absolute risk-aversion, the constraint is more demanding; however, for any situation where $\phi(\mathbf{x})$ is significantly larger than $w(\mathbf{x})$ - which would include most publicly traded companies - $cov(\phi(\mathbf{x}) - w(\mathbf{x}), w(\mathbf{x}))$ will tend to be greater than $Var(w(x))$ and, therefore, the inequality will also tend to be satisfied. Interestingly, for the case of such preferences, the condition can be rephrased as an optimality bound on performance sensitivity

$$\frac{cov(\phi(\mathbf{x}), w(\mathbf{x}))}{Var(w(\mathbf{x}))} \geq 1 + \frac{1}{w(\overline{\mathbf{x}}) - W}, \tag{18}$$

where the left-hand side can be interpreted as the coefficient from a regression of performance on pay.

3 Unbounded Compensation

3.1 Revisiting the First-Order Approach

When effort is discrete, the optimal contract must feature an upper bound $w(\mathbf{x}) \leq w(\overline{\mathbf{x}})$ since, for any $\mathbf{x}$, the optimality condition derived from the firm's Lagrangian is

$$\frac{1}{u'(w(\mathbf{x}))} = A(\lambda + \mu) - \mu\frac{f(\mathbf{x}|0)}{f(\mathbf{x}|1)} \leq A(\lambda + \mu). \tag{19}$$

The binary effort model is therefore incompatible with compensation structures that are unbounded by design, i.e., such that the manager is required to hold a potentially large upside pay.[4] Further, any choice over an effort set in which, in equilibrium, the preferred effort deviation(s) are bounded away from $e = 1$ implies a bounded compensation since we can then relabel the binding deviation as $e_d = 0$ and obtain a first-order condition similar to (19).

To extend the model to unbounded compensation, we assume next that effort $e \in \mathbb{R}^+$ is chosen in a continuous set so that, denoting by convention $e^* = 1$ as the induced effort, (IC) becomes:

$$1 \in \arg\max_e \int A(e)u(w(\mathbf{x}))f(\mathbf{x}|e)d\mathbf{x}, \tag{21}$$

where $A(e)u(w(\mathbf{x})$ is differentiable and decreasing in e. If the first-order approach is valid (Holmström 1979), the above constraint can be replaced by the first-order condition on the maximization:

$$\int u(w(\mathbf{x}))(A'(1) + A(1)\frac{f_e(\mathbf{x}|1)}{f(\mathbf{x}|1)})f(\mathbf{x}|1)d\mathbf{x} = 0. \tag{IC2}$$

Earlier theoretical research has shown that the first-order approach can be problematic in agency problems with a given parametric functional form for the likelihood ratio.[5] However, our question is slightly different in terms of what is available to the researcher to consider the validity of the approach, as we assume the researcher observes an unbounded optimal contract (an endogenous variable in the agency model) but does not know the form of the likelihood ratio.

We argue below that, perhaps surprisingly, the potential technical problems from the first-order approach can be ignored when explaining unbounded compensation. The first-order condition may not be necessary if the optimal deviation is large; for example, Chaigneau, Edmans, and Gottlieb 2019 show that certain classes of agency problems imply that the manager's surplus as a function of e is not concave and the manager prefers a large deviation with binding no-effort $e = 0$. However, if this situation occurs in our setting, the optimal compensation will be (observably) bounded and can be analyzed as in Section 2. More

[4]It is readily verified that boundedness is implied by any model in which the set of effort is finite since equation 19 becomes:

$$\frac{1}{u'(w(\mathbf{x}))} = A(\lambda + \mu) - \mu\frac{f(\mathbf{x}|0)}{f(\mathbf{x}|1)} \leq A(\lambda + \mu). \tag{20}$$

[5]The first-order approach is discussed, among others, by Mirrlees 1999, Rogerson 1985b, Jewitt 1988, and Hemmer 2004. As will become clear from the analysis of contracts that follows, whether the first-order approach is valid is, at the moment, not a testable proposition because the counterfactual distribution of output is not empirically observable. It is also possible, as noted by Chaigneau, Edmans, and Gottlieb 2019, that the continuous effort problem implies a best deviation that is not small (i.e., the binding deviation is no effort), in which case the continuous model will be observationally equivalent to one with binary effort.

generally, locally, the fact that deviations should not involve higher or lower effort than $e = 1$ implies that (IC2) is a necessary condition given any observed optimal unbounded $w(\mathbf{x})$. Indeed, any unbounded compensation requires the firm to consider small deviations, and therefore, the Lagrange multiplier associated with (IC2) must be non-zero (implying that e^* must be a global maximum in the manager's problem).[6]

3.2 Identification

One identification issue in the continuous effort model is that the counter-factual distribution of output $f(\mathbf{x}|0)$ and the manager's utility conditional on effort $u(w;0)$ no longer affect the observed contract and, therefore, these constructs cannot be inferred from observables. As such, we need to re-normalize the measurement of effort to $A(1) = -A'(1)\text{sign}(u(w)) = 1$, so that (IR) becomes

$$\int u(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} \geq u(\underline{w}), \tag{IR2}$$

and $u(w) \equiv u(w;1)$ is now defined in terms of the preference conditional on effort. In this formulation, the effort e is measured such that higher efforts create disutility. The manager surplus is now stated relative to a fixed wage with provision of effort (or equivalently, assuming that the manager acquires supplementary private benefits from shirking).[7]

Proposition 5 *For any $u(\underline{w}) = \beta$, there exists a unique counter-factual distribution*

$$\frac{f_e(\mathbf{x}'|1)}{f(\mathbf{x}'|1)} = \frac{\frac{1}{u'(w(\mathbf{x}'))} - \mathbb{E}(\frac{1}{u'(w(\mathbf{x}))})}{cov(\frac{1}{u'(w(\mathbf{x}))}, u(w(\mathbf{x}))/\beta)} \tag{22}$$

that solves the first-order condition of the firm's problem.

Note that it may still be the case that these first-order conditions are not sufficient if there is another wage profile $w_0(.)$ satisfying these conditions but preferred by the firm. While there is no simple condition to rule this out, the fact that the coefficients are unique implies that, if this occurs, the observed contract cannot be optimal, implying a possible (albeit admittedly tenuous) falsification of the theory. Numerically, this can be verified by directly

[6]The only remaining problem related to the first-order approach occurs if the manager has multiple global maxima, which would change the firm's problem by adding other incentive constraints at different effort levels. Note, however, that this would require a manager surplus with multiple global optima and is typically not generic. Further, empirically, this question is unanswerable absent full (parametric) knowledge of $A(e)$ and $f_e(\mathbf{x}|1)/f(\mathbf{x}|e)$, whereas observations only contain information about these objects around the optimal effort.

[7]For any effort e such that $A'(1) \neq -\text{sign}(u(w))$, one can redefine $e_2 \equiv -A'(1)\text{sign}(u(w))(e-1)+1$ and $A_2(e_2) \equiv A(\frac{-1}{A'(1)\text{sign}(u(w))}(e_2-1)+1)$ and satisfies: $A_2(1) = 1$ and $A_2'(1) = \frac{-A'(1)}{A'(1)\text{sign}(u(w))} = -\text{sign}(u(w))$.

solving the agency problem under (5): if there exists another solution $w_0(.)$ preferred by the firm, the contract $w(.)$ can no longer be rationalized as an optimal contract.[8]

Equation (22) can also be used with supplementary parametric assumptions on $f(.|e)$. The left-hand side of the equation can be specified up to certain unknown parameters of the distribution. For example, if $\mathbf{x}$ is a stock return and follows a geometric Brownian motion. In that case, the board might specify a counter-factual distribution $f(\mathbf{x}|e)$ as a geometric Brownian process with a different drift, implying that $f(.|e)$ may be known up to the drift, which can be fitted from (22). More generally, given a family of distributions such that there is an inverse mapping from the inferred $\frac{f_e(\mathbf{x}|1)}{f(\mathbf{x}|1)}$ to a single distribution, it is possible to recover the entire set of counterfactual efforts. If, in addition, the assumed family of distributions is not too large and does not admit a set of parameter values $\frac{f_e(\mathbf{x}|1)}{f(\mathbf{x}|1)} = G(\mathbf{x})$ for any continuous function $G(.)$, certain risk-aversions may be found incompatible with this parametric assumption.

The marginal effect of effort on output can then be written as

$$\mathcal{M} \equiv \int f_e(\mathbf{x}|1)\phi(\mathbf{x})d\mathbf{x} = \frac{cov(\frac{1}{u'(w(\mathbf{x}))}, \phi(\mathbf{x}))}{cov(\frac{1}{u'(w(\mathbf{x}))}, u(w(\mathbf{x}))/\beta)}, \tag{23}$$

and is greater when the manager's marginal utility covaries more with the firm's surplus than with the manager's utility. However, one problem with this formulation is that the productivity of effort is measured relative to a proportional effect on utility $A'(1)$ normalized to one and is therefore dependent on the specification of the utility function. It is thus more convenient to scale this measure by a monetary cost of effort, similar to the case of binary effort. Specifically, we consider a function

$$u(\underline{w}) = A(e)u(\underline{w} + \mathcal{C}(e)), \tag{24}$$

which represents the fixed payment to be made to the manager equivalent to lower effort. Differentiating both sides of this expression and evaluating it at the induced effort yields

$$\mathcal{C}'(1) = \frac{|u(\underline{w})|}{u'(\underline{w})} = \frac{|\beta|}{u'(u^{-1}(\beta))}, \tag{25}$$

which represents the compensating differential to sustain a small increase in effort. Then, we define the relative productivity of effort (i.e., per unit of monetary cost) as

[8] This would be the only restriction to falsify the model, as condition (17) from Gayle and Miller 2015, which compares the firm's utility in the contract versus without effort is no longer computable: for a small deviation, we can no longer observe the off-equilibrium cost to the firm of inducing a different action profile.

$$\Delta_c = \frac{\int f_e(x|e)\phi(\mathbf{x})d\mathbf{x}}{\mathcal{C}'(1)} = \frac{cov(\frac{u'(u^{-1}(\beta))}{u'(w(\mathbf{x}))}, \phi(\mathbf{x}))}{cov(\frac{1}{u'(w(\mathbf{x}))}, |u(w(\mathbf{x}))|)}. \tag{26}$$

Finally, as for binary effort, it is possible to characterize the model near risk-neutrality. One obtains the following characterization by taking the limit as the utility function becomes linear and using the limits in earlier Propositions.

Proposition 6 *Under HARA preferences with $a > 0$ finite,*

$$\lim_{r\to 1} \frac{f_e(\mathbf{x}'|1)}{f(\mathbf{x}'|1)} = \frac{\ln(w(\mathbf{x}')) - \mathbb{E}(\ln(w(\mathbf{x})))}{cov(\ln(w(\mathbf{x})), w(\mathbf{x})/W)}, \quad \lim_{r\to 1} \Delta_c = \frac{cov(\phi(\mathbf{x}), \ln(w(\mathbf{x})))}{cov(w(\mathbf{x}), \ln(w(\mathbf{x})))}. \tag{27}$$

and, under constant absolute risk-aversion $u(w) = -e^{-aw}$, $\lim_{a\to 0} \frac{|f_e(\mathbf{x}'|1)|}{f(\mathbf{x}'|1)} = \infty$ diverges for any $w(\mathbf{x}) \neq W$ and

$$\lim_{a\to 0} \Delta_c = \frac{cov(\phi(\mathbf{x}), w(\mathbf{x}))}{Var(w(\mathbf{x}))}. \tag{28}$$

3.3 Semi-parametric Analysis of Risk Aversion

While Proposition 5 may suggest that only limited knowledge may be gained in the context of small deviations, such small deviations may suggest other natural restrictions on plausible risk-aversion coefficients. Note first that because $f_e(\mathbf{x})$ must change signs, there exists $\hat{\mathbf{x}}$ such that $f_e(\hat{\mathbf{x}}|1) = 0$. In what follows, we make the following assumption

Assumption 1 *There exists a function $h(\mathbf{x}, e)$, where $h_e(\mathbf{x}, 1) \neq 0$, such that*

$$f(\mathbf{x}|e) = g(h(\mathbf{x}, e)) \tag{29}$$

As a special case, assumption 1 will hold over classes of agency problems where the manager does not increase the riskiness of the performance measure, as defined by $x = \xi(e) + u$, where $\xi(.)$ is a deterministic function and u is a random variable with density f^u. Then, $f(x|e) = f^u(x - j(e))$.[9] This linearity restriction is satisfied in models with normal noise, e.g., among others, Feltham and Xie 1994, Holmstrom and Milgrom 1991, Liang, Rajan, and Ray 2008, or Dutta and Fan 2014.[10]

[9] While this restriction may not always hold, one should note that, in the contracting problem that we analyze, the firm is forming priors about the effect of effort $f(x|e)$ that cannot be confirmed empirically for off-equilibrium efforts $e \neq 1$. It may be ambiguous whether effort should serve to increase risk (the manager expands the firm toward better but also more volatile investments) or decrease risk (the manager can identify better projects with low downside risk). Conceptually, it may thus seem natural to calibrate the distribution of effort in a manner that does not involve beliefs about changing distributions.

[10] The focus of this literature is, however, different from ours. It typically assumes CARA utilities and linear

Corollary 1 *If assumption 1 holds, $\hat{\mathbf{x}}$ must be a peak of $f(\mathbf{x}|1)$. Then, the manager's preference must satisfy:*

$$\int \frac{1}{u'(w(\mathbf{x}))} f(\mathbf{x}|1) d\mathbf{x} = \frac{1}{u'(w(\hat{x}))}. \tag{30}$$

If the empirical distribution of x is single-peaked, which will hold under log-concavity (Bagnoli and Bergstrom 2005), $\hat{x}$ is uniquely recovered from knowledge of the observed $f(.|1)$. Equation (30) can be used as an additional robustness check on the risk-aversion, noting that the observed $w(\hat{x})$ will often restrict valid risk-aversion coefficients. Interestingly, under certain circumstances, this property can imply that risk-neutrality is the only preference compatible with some wage structures.

4 Application to Tesla's 2018 Incentive Plan

4.1 Institutional Background

We apply the model to the 2018 Tesla incentive plan and the recent court decision in Tornetta v. Musk (McCormick 2024). Richard Tornetta (the plaintiff) filed suit that the board of Tesla *"breached their fiduciary duties by awarding Elon Musk a performance-based equity-compensation plan"* that was *"250 times larger than the contemporaneous median peer compensation plan and over 33 times larger than the plan's closest comparison"*. Under the terms of the incentive plan, which was approved on March 21, 2018, with 73% of all common shares not owned by Musk voting in favor (Tesla 2018), Musk was to receive up to $20,264,042$ options to buy shares at a price \$350.02 exercisable over a 10-year period. This was divided into 12 equal-size tranches roughly equal to 1% of the total number of shares such that each new tranche must meet a (i) market capitalization threshold and (ii) operational milestone, as stated in Table 1 below. The fair value of the plan was estimated by Tesla to be \$2.615 billion, assuming a risk-free rate of 2.64%, a volatility of 45.35%, and an illiquidity premium of 10.63%. By the end of 2023, Tesla's market capitalization had increased over 14-fold, and Musk had achieved 12 operating milestones, implying a maximum value of \$54.8 billion at the market price such that all equity tranches vest.

The court opined that, because of a position as *"Superstar CEO"* and his 21.9% equity stake, Musk effectively controlled Tesla, a determination undisputed by the defense. While Delaware law may allow defendants to shift the burden of proving fairness to the plaintiff,

contracts, and its main interest is multi-tasking problems. While multi-tasking, i.e., a multi-dimensional effort choice e, opens interesting issues, it is non-trivial to identify in this model without placing a special structure on the type of multi-tasking considered. Unfortunately, identifying the parameters of interest is difficult even with a single task, and we are not yet sure how to give evidence of which multi-tasking problem is being solved.

the court rejected the applicability of this provision, noting that shareholders were inaccurately informed about the personal relationships between Musk and key directors and the feasibility of the milestones. The court ultimately sided with the plaintiff, voiding the incentive payment. As of May 1, 2024, the Tesla board resubmitted the incentive payment to a shareholder vote on an ex-post basis.

This case has been widely interpreted as a landmark decision requiring boards to provide a more explicit description of the assumptions made to justify unusual compensation structures; to this effect, our methodology provides a simple approach to evaluate the primitives required to explain a compensation structure. To better calibrate the model, we note several key facts raised in the post-trial opinion below.

(i) Musk noted in a conference call that he did not intend to be *"CEO forever"* but also clarified *"I intend to be actively involved with Tesla for the rest of my life. Hopefully, stopping before I get too old – or too crazy, I don't know. But essentially for as long as I can positively contribute to Tesla, I intend to be–to have a significant involvement with Tesla"* (May 3, 2017, earnings call). In court testimony, Musk unequivocally stated that *"he would have remained at Tesla even if stockholders had rejected a new compensation plan."* (McCormick 2024, p.38)

(ii) Board minutes show that the incentive plan would ensure continued focus on innovation but did not mention the incentive role of Musk's pre-existing 21.9% equity stake and why such ownership (substantially greater than peers) would be insufficient to align incentives. As part of incentive alignment, the board discussed the opportunity costs of Musk devoting time to Tesla versus other interests, although Musk testified against this concern (*"that would be silly"*, McCormick 2024, p.59).

(iii) The operating milestones were based on ratios of large US peer companies. Further, in July 2017, the Board privately made projections that implied that *"three of the revenue milestones and all of the adjusted EBITDA milestones would be achieved in 2020"* and the court noted that, while unknown to shareholders, achieving these three milestones was highly probable (70%). These forecasts were not shared in the proxy statement before the vote, and suggested to the court that the market capitalization thresholds subsumed short-term operating milestones. Documents submitted by the defense included the Monte Carlo simulation used to assess the grant's fair value. The first tranche would vest with a probability of 45.55%, falling below 10% for the sixth tranche onward.[11] The valuation did not account for the operational milestones or the private forecasts.

[11] Unfortunately, the court case has been classified as confidential, limiting access to parties directly in-

(iv) The option exercises required a 5-year holding period, which was used to discount the grant's fair value by a 10.52% illiquidity discount. However, Musk could still effectively divest from the new shares obtained from the option exercises by selling any of his prior shares. The board also discussed the need for this holding period to allow clawback provisions if accounting metrics were later restated (as restatements can occur 3 to 5 years after the financial statements are released, see Bertomeu et al. 2021).

The defense disputed three main arguments made by the plaintiff: first, the lack of transparency prior to the minority shareholder vote and whether the private information exchanged materially affected the contract; second, the need for incentives above and beyond those provided by the equity holdings; and third, the excessive bargaining power of Musk vis-a-vis minority shareholders and whether the board may have been negotiating on behalf of Musk. The methodology from previous sections allows us to quantify each of these assertions by inverting the contract to make its assumptions explicit and interpretable. To the first question, we analyze whether the operating milestones materially affected the contract. To the second question, we can evaluate the implicit assumption about the opportunity cost to Musk and the anticipated contribution of effort to the firm value. To the third question, we can recover the surplus paid, net of opportunity costs. We use this application as an example because it presents the benefit of incorporating rich institutional content, but the main insights carry over to other superstar compensation structures.

Formally, Musk repeatedly argued that the compensation was not solving a retention problem, which may be descriptive of more general environments where leaving the company would involve a significant disutility. For this reason, we interpret $\underline{w} - W_0$, where W_0 is the market value of the equity owned as of the date the incentive contract was offered, as bargaining power.[12] We also note that the board explicitly noted concerns that Musk may dissipate his attention without an appropriate plan, and prior research has argued that Superstar CEOs may no longer maximize shareholder value or under-perform (Malmendier and Tate 2009); the board concerns were partly vindicated by Musk's continuing engagements in other projects (including SpaceX and Twitter/X) as well as an active social media presence. Hence, this suggests a model in which the contract was designed to compensate Musk for not engaging in alternative activities of personal benefit to him but not necessarily to Tesla.

volved in the lawsuit. These numbers are from the post-trial opinion, selected from the documents by the court.

[12] We believe that this assumption is plausible for most Superstar CEOs since they likely receive personal benefits – reputational, psychological, or perks – from leading the company they often founded or joined in its early stages, and it would be very difficult to imagine that CEOs that have reached Superstar status, such as Jeff Bezos, Steve Jobs or Elon Musk, retire toward a no-effort position, or move to a competitor because they are not receiving enough pay.

4.2 Model Calibration

The compensation structure $w(\mathbf{x})$ for Elon Musk's 2018 incentive plan is given in Table 1, to which we add the 21.9% equity stake. We assume that the board calibrated the payout of \$54.8 billion, also referred to as "maximum" payout in Tesla 2018 and the post-trial opinion (McCormick 2024), as indicative of the desired action $e = 1$, i.e., $\overline{\mathbf{x}}$ is set such that the market capitalization is \$650 billion.

In the benchmark model, the process to recover the primitives of the agency problem requires a calibration of the maximum wage $w(\overline{\mathbf{x}})$ which, consistent with our earlier discussion, we interpret the payment received for achieving all 12 tranches. However, one difficulty is that we do not know when Musk would exercise his options (or, more generally, how a superstar CEO manages their portfolio), which may be affected by personal preferences for owning shares or considerations of illiquidity from selling a large number of shares in the open market (i.e., over 30% of the equity conditional on the maximal payout). Another difficulty is that due to the existence of operational milestones, achieving a market capitalization above $\overline{\mathbf{x}}$ does not necessarily result in the vesting of all 12 tranches. To address these issues, we set $w(\overline{\mathbf{x}})$ equal to the expected value of the equity portfolio, including all vested options and initial equity stake, by taking expectations over realized events with market capitalizations above \$650 billion.[13] Put differently, this is equivalent to assuming that, for contracting purposes, the board does not differentiate between realizations of $\mathbf{x}$ once $\overline{\mathbf{x}}$ has been reached. Note that, in the theoretical framework, the additional performance sensitivity in this range no longer has any incentive value, and, in the risk-neutral limit, the cost to Musk for holding additional risk is small. Finally, we express all quantities in present values (as of the contracting date) and in pre-tax dollars.[14]

We model the compensation signal $\mathbf{x}$ as the path of a continuous-time process for prices p_t, revenue rev_t, and EBITDA, eb_t, using the following hierarchical process:

[13] For example, if 12 out of the 16 accounting vesting conditions have been met before maturity and the market capitalization at maturity is above \$650 billion, Musk receives the expected payout of 58.1 billion shares (37.8 billion shares from his pre-existing stake and 20.3 billion shares from the vesting of all 12 tranches) conditional on the market capitalization being above \$650 billion. If only 11 of the 16 accounting vesting conditions have been met, Musk receives the expected payout of 56.4 billion shares (only 11 tranches or 18.6 billion shares would vest).

[14] Unlike the GAAP stock option expense, which is valued ex-ante based on the fair value of the options when issued and then amortized over the maturity of the option, the tax implications of option payouts are assessed on the ex-post difference between the value of the shares and the strike at the date of exercise Graham, Lang, and Shackelford 2004. Under the (admittedly reductive) simplification of a linear firm pre-tax income of $\tau_c = 21\%$ as of 2024 and a personal marginal income tax of $\tau_p = 37\%$ at the highest brackets, excluding State and local taxes. In after-tax dollars, we then need to replace $w(\mathbf{x})$ by $(1 - \tau_c)w(\mathbf{x})$ and $u(w(\mathbf{x}), e)$ by $u((1-\tau_p)w(\mathbf{x}), e)$. Then, dividing all terms in the firm's problem by $(1-\tau_c)$, the main analysis is equivalent to rescaling the manager's payoff by $(1 - \tau_p)/(1 - \tau_c)$ in after-tax dollars.

Market Capitalization Milestone	Operational Milestone	# Options
$ 100 B	1 / 16	1,688,670
$ 150 B	2 / 16	3,377,340
$ 200 B	3 / 16	5,066,010
$ 250 B	4 / 16	6,754,680
$ 300 B	5 / 16	8,443,350
$ 350 B	6 / 16	10,132,021
$ 400 B	7 / 16	11,820,691
$ 450 B	8 / 16	13,509,461
$ 500 B	9 / 16	15,198,031
$ 550 B	10 / 16	16,886,701
$ 600 B	11 / 16	18,575,371
$ 650 B	12 / 16	20,264,042

(a) Vesting Requirements

Revenue Milestones	EBITDA Milestones
$ 20 B	$ 1.5 B
$ 35 B	$ 3 B
$ 55 B	$ 4.5 B
$ 75 B	$ 6 B
$ 100 B	$ 8 B
$ 125 B	$ 10 B
$ 150 B	$ 12 B
$ 175 B	$ 14 B

(b) Operational Milestones

Source: Proxy Statement (DEF 14A), February 8, 2018
Note: This table summarizes the compensation structure offered to Elon Musk on January 21, 2018. The options vest in tranches if the market capitalization and the required number of operational milestones (out of 16) are met. Market capitalization refers to the six months or thirty days prior to and including the determination date (vesting). Operational milestones refer to the sum of the four consecutive quarters prior to the determination date.

Table 1: Compensation Structure

$$
\begin{cases}
dp_t = \mu_p p_t dt + \sigma_p p_t dW_t^p \\
d\dfrac{rev_t}{p_t} = \mu_r dt + \sigma_r dW_t^r \\
d\dfrac{eb_t}{rev_t} = \mu_e dt + \sigma_e dW_t^e,
\end{cases}
\tag{31}
$$

where rev_t and eb_t represent the revenue and EBITDA flows accumulated over the past four quarters, as defined in the contract. Tesla's disclosures and historical information can inform several parameters of these processes. Further, the board anchored its revenue assumptions based on multiples, and the model of adjusted EBITDA in terms of margins allows us to generate processes that start near zero and then expand quickly as revenue scales. We simulate the processes monthly, and the vesting conditions are computed following the conditions stated in Table 1. The calibration of this process is given in Table A.1 with details in Appendix A.

4.3 Benchmark Contract Parameters

We examine the underlying contract parameters in the benchmark of near risk-neutrality, following Propositions 2 and 3 in the case of HARA and CARA utilities, respectively, and under logarithmic utility from Proposition 1. We further define the surplus of the principal as:

$$
\Delta_p \equiv \mathbb{E}(\phi(\mathbf{x}) - w(\mathbf{x})|e = 1) - (\mathbb{E}(\phi(\mathbf{x})|e = 0) - \underline{w}), \tag{32}
$$

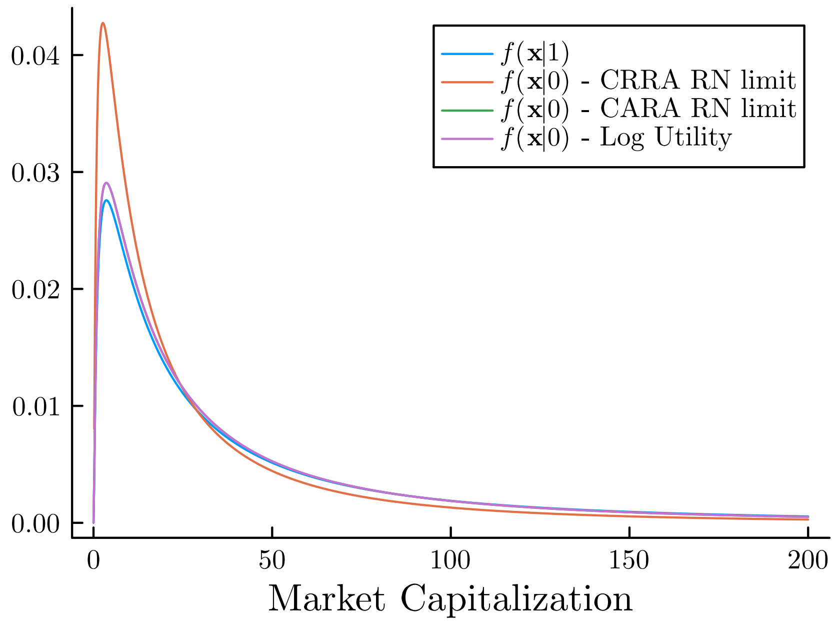
where the first (second) term represents the expected profits when the manager exerts effort (no effort) and $\phi(\mathbf{x})$ is defined as the market capitalization.[15] Recall that the manager surplus is defined as:

$$
\Delta_m \equiv \underline{w} - W_0, \tag{33}
$$

where W_0 is the value of the equity stake owned by the manager at the start of the grant. Elon Musk owns 21.9% of Tesla's shares worth $\$60.56B$ at the start of the grant, so $W_0 = \$13.25B$.

Figure 1 plots the distribution of stock prices conditional on effort $f(\mathbf{x}|1)$ and no effort $f(\mathbf{x}|0)$. As expected, the counterfactual distributions under no effort exhibit lower means and thinner upper tails, reaching zero at $\overline{\mathbf{x}}$, above which the performance measure can only

[15]Note that the reservation wage $\underline{w}$ is still paid by the principal to the manager in the case of no effort because the principal still employs the manager at a fixed wage equal to the manager's reservation utility.



Plots the densities of the market capitalization conditional on effort and no effort for the binary effort model, for CRRA and CARA utilities assuming that the manager is near risk-neutrality, and for the Log utility. The density conditional on effort $f(\mathbf{x}|1)$ is derived from the calibration parameters. The densities conditional on no effort $f(\mathbf{x}|0)$ are estimated from equation 2. This plot only considers market capitalization milestones. $f(\mathbf{x}|0)$ for CARA and Log utility are overlapping.

Figure 1: Densities conditional on effort for the binary effort model.

Description	CRRA RN limit	CARA RN limit	Log Utility
Reservation wage ($\underline{w}$)	\$ 6.54 B	\$ 9.99 B	\$ 4.14 B
Cost of Effort ($\mathcal{C}$)	\$ 8.36 B	\$ 4.91 B	\$ 0.57 B
Contribution of effort to output (Δ)	\$ 30.86 B	\$ 16.74 B	\$ 16.74 B
Firm surplus (Δ_p)	\$ 22.5 B	\$ 11.83 B	\$ 5.98 B
Manager surplus (Δ_m)	\$ -6.71 B	\$ -3.26 B	\$ -9.11 B

Note: Tabulates the contract estimates for CRRA and CARA utilities, assuming that the manager is nearly risk-neutral, and for logarithmic utility. These estimates are derived from the binary effort model presented in section 2.

Table 2: Estimates near risk-neutrality for the binary effort model.

occur under effort. As noted in the theoretical model, the counterfactual distributions under no effort are identical in the risk-neutral limit of a CARA preference and for a logarithmic preference – however, the remaining parameters differ due to differences in the concavity of the preference. In particular, the reservation wage is much smaller with risk-aversion because holding the contract fixed entails an additional disutility from holding risk. This also implies, throughout this analysis, that the risk-neutral benchmarks tend to be consistent with more manager surplus. Other parameters, while they depend on the preference, are all within the same order of magnitude.

Table 2 tabulates the contract parameters. Our first insight is that, near risk-neutrality for CRRA and CARA utilities and for a logarithmic utility, the firm surplus is positive, between \$6B to \$23B (or between \$31B and \$17B before netting out Musk's compensation and equity), consistent with the difficulty in ruling out an optimal contract for sufficiently low risk-aversion. Even in the case of Musk's incentive contract, which is unique in featuring magnitudes in the variability of pay similar to shareholder value, we cannot rule out an optimal contract near risk-neutrality even under CARA. Interestingly, these inferred parameters also suggest that the ex-ante expected contribution of effort is only a small fraction of the realized ex-post growth in Tesla's market capitalization (\$790B on Dec 31, 2023, versus \$60B at the date of the contract), which would imply that much of the realized performance would have been due to random factors other than effort.

Our second insight is that the contract was designed with the assumption of a substantial cost of effort. Based on our baseline parameters, the inferred monetary cost of effort $\mathcal{C}$ is at \$8B under CRRA and around \$5B for CRRA. This parameter is smaller under logarithmic utilities, which stems from a similar intuition as given earlier as part of the pay serves to compensate for risk rather than effort – but even then, it remains at \$600 million. Thus, we note that the observed compensation is rationalized as a very large risk premium if Musk were risk-averse or, perhaps more plausibly, to compensate for the opportunity cost of alternative occupations, consistent with board discussions. Note that these magnitudes, while large, are consistent with the valuation of Musk's other entrepreneurial ventures.

The third insight is perhaps the most surprising. After netting out the existing share ownership, the manager surplus is *negative* for all specifications, and this effect is stronger with higher risk-aversion. In other words, Musk would be wealthier in a counterfactual where all existing ownership was sold. Assuming that the board correctly calibrated the cost of effort, this finding is thus incompatible with Musk exerting excessive bargaining power. This could be due to two interpretations. First, Musk may be achieving non-monetary benefits from leading Tesla, which hurts his bargaining power by reducing the outside option of leaving the firm. Second, it could be the case that if Musk is perceived by the market as essential to Tesla (even absent high effort), leaving the firm would reduce Tesla's stock price and reduce the value of his existing shares below the market price at the date of the contract; this, again, serves to reduce bargaining power. To set ideas, if Musk had no bargaining power and were paid his outside option, the required effect on Musk's 21.9% ownership for him to accept the contract (versus leaving) would be a decrease in Tesla's stock price by 50% under CRRA, 24% under CARA, and nearly 70% under logarithmic utility – of course, this would be lower under the earlier hypothesis of non-monetary benefits.[16]

4.4 Other Risk-aversions

In this section, we estimate the contract parameters for risk aversions other than logarithmic preferences, where the horizontal axis captures increasing aversion to risk. The inferred cost of effort $\mathcal{C}$ decreases in the hypothesized risk-aversion. This may seem, at first sight, surprising because agency theory predicts that it is more costly to the principal to induce effort when the required risk premium is higher. However, throughout these analyses, note that these are not theoretical comparative statics. Instead, the observed contact is held constant, and the inferred parameter is a primitive (or exogenous coefficient in a theoretical model). A heuristic interpretation of this result is that, when factoring in a higher risk premium, the share of the contract payments covering the cost of effort must be smaller. The same logic is at play for the manager's reservation wage, implying an inferred reservation wage decreasing in risk-aversion. As the risk premium explains an increasing share of total payments, less of this surplus remains as residual surplus to the manager.

The inferred contribution of effort to output also decreases in risk-aversion. This may again seem counter-intuitive because, given the greater cost of compensating the manager, we might have expected effort to be more important for the principal when holding the

[16] The value of Musk's 21% ownership is $13.25B.

compensation arrangement fixed. In other words, a principal choosing to incur a large risk premium should consider effort important. However, recall that the inferred monetary cost of effort also decreases in risk-aversion, so we are comparing the same effort costs as all parameters are being simultaneously readjusted. Indeed, the counter-factual distribution conditional on no effort becomes closer to $f(.|1)$ when the manager is more risk-averse.

4.5 Parametric Model

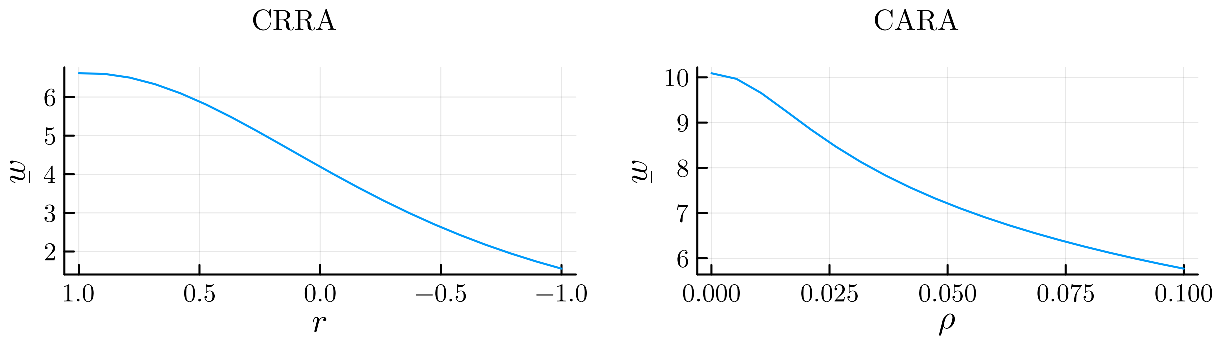
While the baseline model offers estimates conditional on risk-aversion being known, it is usually rationalizable with many levels of risk-aversion. In the case of Musk, the firm surplus is positive for risk-aversion up to $r = -1.25$ and $\rho = 0.25$ under CRRA and CARA, corresponding to certainty equivalents of \$9.8 and \$2.8 for a gamble paying zero or \$2B with equal chance. Next, we explore a parametric implementation of the model, which involves an intuitive representation of the price process in terms of a geometric random walk. This will allow us to evaluate whether near-risk-neutrality is compatible with this type of process or if it would require an ad-hoc specification of the board's beliefs.

Specifically, we parameterize the counterfactual distribution $f(\mathbf{x}|0)$ as a log-normal distribution representing the 10-year ahead distribution of a geometric Brownian motion with drift μ_0 and volatility σ_0. We then recover the level of risk-aversion (r for CRRA and ρ for CARA utility) and counterfactual distributions that satisfy the theoretical restrictions of the contracting problem. Unlike in the non-parametric benchmark approach, this problem is over-identified because the parametric model imposes a functional form on the likelihood $f(\mathbf{x}|0)/f(\mathbf{x}|1)$, up to the drift and volatility of the distributions. For $f(\mathbf{x}|1)$, we set the drift to the risk-neutral measure r_f and the volatility to the calibrated value reported in the proxy statement and fit the remaining parameters $(\hat{\xi}, \hat{\mu_0}, \hat{\sigma_0})$ by minimizing the distance from the theoretical likelihood ratio:

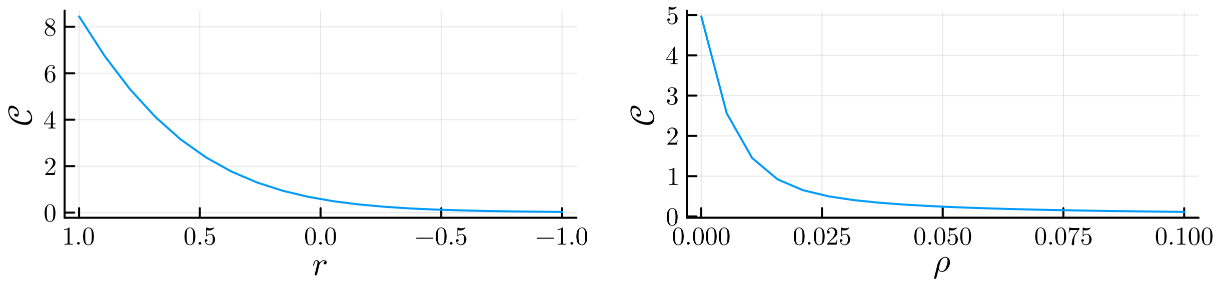
$$(\hat{\xi}, \hat{\mu_0}, \hat{\sigma_0}) = \arg \min_{\xi, \mu_0, \sigma_0} \int \left(\frac{f(\mathbf{x}|\mu_0, \sigma_0)}{f(\mathbf{x}|r_f, \sigma_p)} - \frac{\frac{1}{u'(w(\overline{\mathbf{x}}))} - \frac{1}{u'(w(\mathbf{x}))}}{\frac{1}{u'(w(\overline{\mathbf{x}}))} - \mathbb{E}(\frac{1}{u'(w(\mathbf{x}))})} \right)^2 f(\mathbf{x}|r_f, \sigma_p) d\mathbf{x}, \tag{34}$$

where ξ is the risk-aversion parameter ($\xi = r$ under CRRA utility and $\xi = \rho$ under CARA utility).

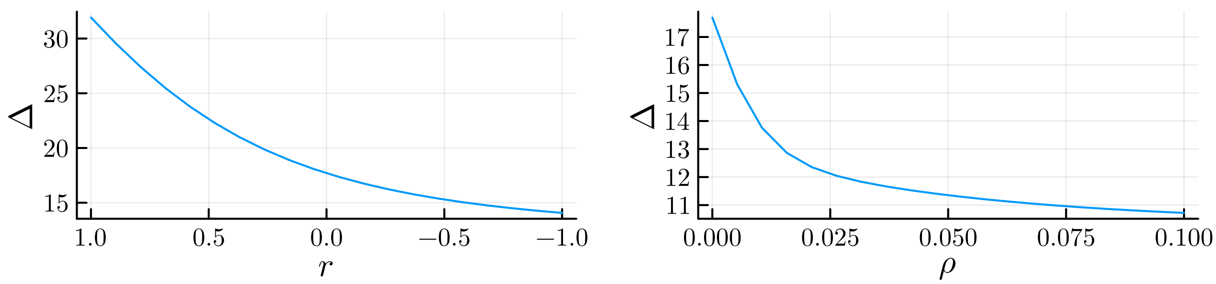
Table 3 reports the drift and volatility of the counterfactual distributions, risk-aversion, and the corresponding contract parameters for both CRRA and CARA utility functions. Interestingly, near-risk-neutrality and the associated counterfactual distributions' parameters best fit an assumed geometric Brownian motion. In Figure 3, the parametric counterfactual



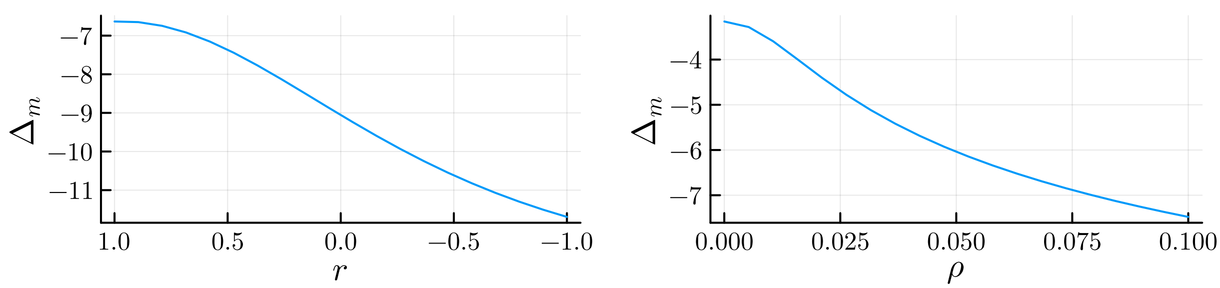
(a) Reservation wage $\underline{w}$



(b) Cost of effort C



(c) Contribution of effort to output Δ



(d) Manager surplus Δ_m

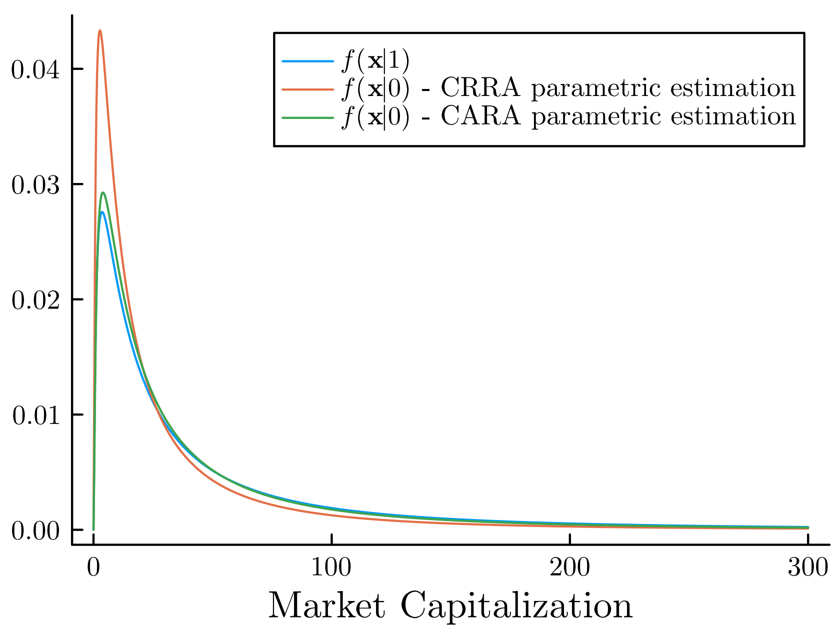
Plots the contract estimates as a function of risk-aversion for the binary effort model presented in section 2. The left (right) plots show the estimates under CRRA (CARA) utility. These estimates consider the full vesting conditions (market capitalization and operational milestones) using the calibrated parameters from table A.1. The manager tends toward risk-neutrality as $r \to 1$ for CRRA and as $\rho \to 0$ for CARA.

Figure 2: Estimates for the binary effort model as a function of risk-aversion.

Description	CRRA	CARA
Risk-aversion parameter (r/ρ)	1.0	0.0
Drift under no effort (μ_0)	-0.034	0.005
Volatility under no effort (σ_0)	0.43	0.43
Reservation wage ($\underline{w}$)	\$ 7.8 B	\$ 9.96 B
Cost of Effort ($\mathcal{C}$)	\$ 6.97 B	\$ 4.82 B
Contribution of effort to output (Δ)	\$ 27.0 B	\$ 11.16 B
Firm surplus (Δ_p)	\$ 20.06 B	\$ 6.34 B
Manager surplus (Δ_m)	\$ -5.45 B	\$ -3.29 B
Distance from theoretical LR	0.00186	0.00657

Note: Tabulates the contract estimates based on a parametric estimation of $f(\mathbf{x}|0)$ for the binary effort model. The last row shows the distance between the parametric and the theoretical likelihood ratio, as expressed in equation 34.

Table 3: Parametric estimates for the binary effort model



Plots the densities of the market capitalization conditional on effort or no effort for CRRA and CARA utility functions. Densities conditional on no effort ($f(\mathbf{x}|0)$) are parametrically estimated.

Figure 3: Densities conditional on effort from the parametric estimation.

densities under no effort are very similar to the non-parametric estimates found in the risk-neutral limit for CRRA and CARA utility functions.

4.6 Contract Complexity

In practice, executive contracts exhibit significant complexity in the form of multi-dimensional performance measurement, performance and time-vesting conditions, clawbacks, and ownership guidelines, among other examples. In the court case against Tesla, this complexity was found to have been potentially misleading to shareholders, as shareholders may not have been aware of the near-term feasibility of operational milestones. However, while part of a larger debate about price versus non-price compensation, it is possible in our setting to evaluate if a substantially simpler contract involving fewer variables or a structure more familiar to shareholders (e.g., a linear surplus-sharing with straightforward equity grants) would have achieved very similar results. If so, it might be argued that the inclusion of complexity in contracts constitutes unnecessary window-dressing as incentive payments that are unnecessary or obfuscate total compensation, especially given that transparency about the wage function or the distribution of performance measures under non-price variables is much lower. Answering this question, however, requires a counterfactual where the contract is re-optimized with a simpler structure.

4.6.1 Complexity due to Operational Milestones

We first revisit the problem of operational milestones and, more generally, whether the inclusion of the accounting-based variables had any material effect on incentives. Interestingly, as noted in court documents, these milestones were originally not part of the discussion between the board and Musk and were later introduced to address potential concerns with specifying the compensation exclusively on price. Further, these milestones significantly raise the complexity of the contract, because they feature a complex set of conditions (such as picking incremental milestones over two lists), and do not have common knowledge distributions relying on market prices. It may also be a concern that subjectivity regarding how these milestones are modeled could affect our inferred parameters.

In table 4, we show that the operational milestones have minimal impact on the inferred parameters to the extent that a contract without these milestone conditions implies almost identical estimates. In our baseline calculations, we did not include private information indicating that the first milestones were likely to be satisfied in the near term, thus further suggesting that knowledge about the milestones had little bearing on the contract.

To explain this finding, Table 5 shows that the issue is not about the first milestones being

Utility	CRRA RN limit		CARA RN limit		LOGU	
Description	Market	Market & Op.	Market	Market & Op.	Market	Market & Op.
Reservation wage ($\underline{w}$)	\$ 6.56 B	\$ 6.55 B	\$ 10.01 B	\$ 9.99 B	\$ 4.16 B	\$ 4.15 B
Cost of Effort ($\mathcal{C}$)	\$ 8.57 B	\$ 8.46 B	\$ 5.12 B	\$ 5.03 B	\$ 0.58 B	\$ 0.58 B
Contribution of effort to output (Δ)	\$ 31.6 B	\$ 31.36 B	\$ 17.51 B	\$ 17.29 B	\$ 17.51 B	\$ 17.29 B
Firm surplus (Δ_p)	\$ 23.03 B	\$ 22.9 B	\$ 12.39 B	\$ 12.27 B	\$ 6.54 B	\$ 6.43 B
Manager surplus (Δ_m)	\$ -6.69 B	\$ -6.7 B	\$ -3.24 B	\$ -3.26 B	\$ -9.09 B	\$ -9.1 B

Note: Compares the contract estimates without and with considering the operational milestones for CRRA and CARA utilities (columns 1 through 4), assuming that the manager is nearly risk-neutral, and for the Log utility (last two columns).

Table 4: Impact of operational milestones on contract estimates for the binary effort model.

satisfied but because most operational milestones were almost surely satisfied conditional on meeting the market capitalization conditions. Across all tranches, the probability that a tranche does not vest because of a failure to meet operational milestones sits below 2%, and they start becoming important only from tranches nine and above, at a modest 25% of not meeting the milestone if the market capitalization is met. While this can affect the value received in the contract, this occurs in the upper tail of realized events and does not significantly affect the inferred parameters or the payments received by Musk. In summary, our counterfactual analysis shows that milestones represented unnecessary complexity and did not serve a material purpose in the contract, to the extent that the firm could have written a nearly equivalent contract using price only.

4.6.2 Complexity due to options

We are now studying the possibility of providing Musk with a simpler stock-based contract. We define a *simple contract* as a contract that relies exclusively on stock-based compensation and is linear in Tesla's market value. Such compensations are widely used in practice and typically vest after a certain amount of time (time-based grants). We ask how costly it would be for Tesla to induce effort via a simple linear contract relative to the actual observed performance-based option (convex) contract. Note that our approach captures the potential benefits of convexity if the likelihood ratio should optimally encourage risk-taking.

To answer this question, we solve for a counterfactual by restricting the contract set to contracts where Musk only receives additional shares. The optimal such contract is the minimum share ownership such that (IC) and (IR) are satisfied.

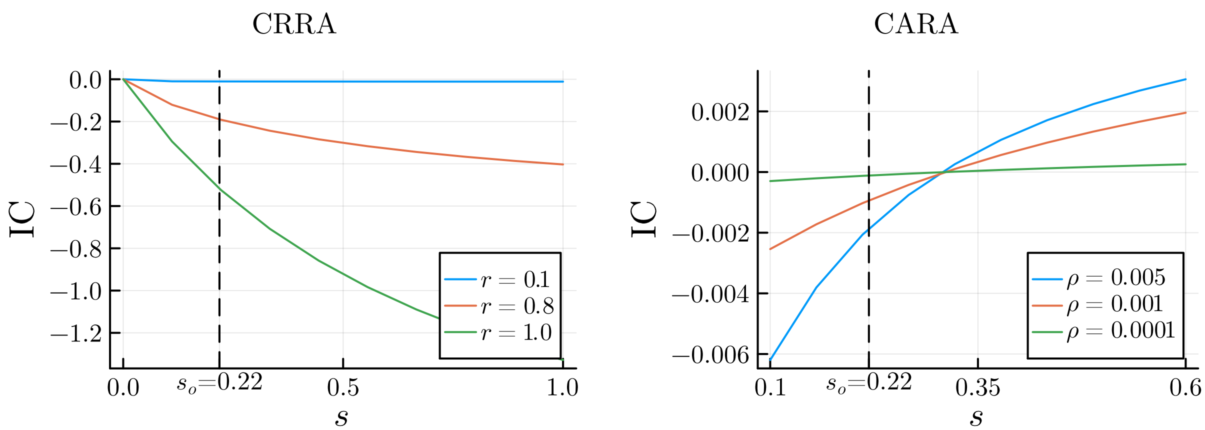
Figure 4 plots these constraints for both CRRA and CARA utility functions. By construction, a positive value indicates that the constraint is satisfied, and a negative value indicates that it is violated. As the contract parameters recovered from the observed performance-based contract depend on the level of risk-aversion, different levels are shown.

A striking observation is that when the manager has a CRRA utility function, no amount of stock ownership, regardless of risk-aversion, elicits effort, and the (IC) is always violated.

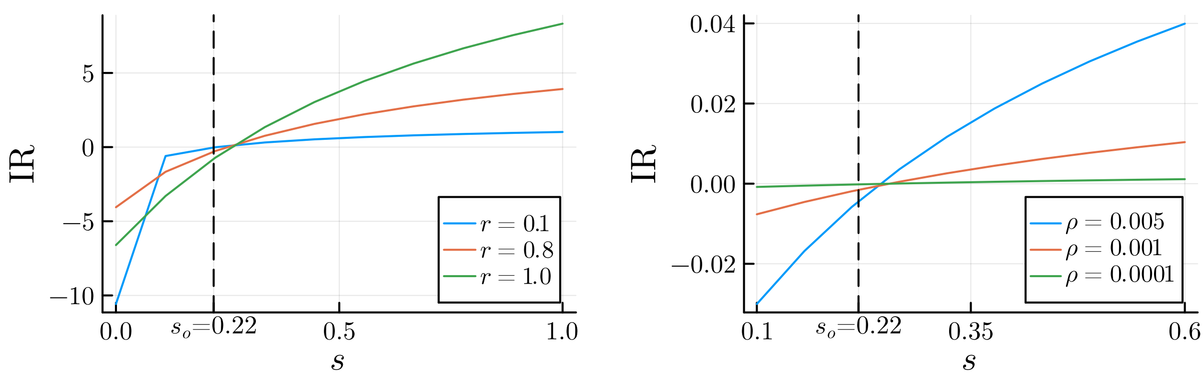
Tranche	Market	Market & Operational	Operational not met
1	0.59	0.59	0.0
2	0.4	0.4	0.0
3	0.3	0.3	0.00015
4	0.23	0.23	0.0
5	0.18	0.18	0.00031
6	0.15	0.15	0.0
7	0.12	0.12	0.0
8	0.1	0.1	0.0
9	0.087	0.072	0.24
10	0.075	0.062	0.27
11	0.065	0.054	0.26
12	0.057	0.047	0.17
Overall			1.8%

Note: The first two columns tabulate the probability that a given tranche will vest under the market capitalization (column 1) or both market capitalization and operational milestones (column 2). The third column tabulates the probability that a given tranche will not vest due to a failure to meet the operational milestones (conditional on meeting the market capitalization milestones). The "Overall" row tabulates to probability across all tranches. Probabilities are computed by simulating 100,000 firms using the calibration parameters from table A.1.

Table 5: Effect of operational milestones on vesting



(a) Incentive compatibility constraint



(b) Individual rationality constraint

Plots the incentive compatibility (IC) and individual rationality (IR) constraints for all possible linear contracts, using estimates from the observed contract, for CRRA and CARA utility function and different levels of risk-aversion. s represents the proportion of the firm the manager owns (linear contract). s_o is the existing ownership of Elon Musk (in proportion to the firm) at the start of the 2018 contract. For a given level of risk-aversion (r or ρ), the contract estimates using the observed performance-based contract are computed and used to evaluate the IC and IR constraints under a linear contract that would provide Elon Musk with a fraction s of Tesla's shares. Positive values for the constraints indicate that they are satisfied. Negative values indicate a violation. The contract estimates are based on the binary effort model presented in section 2.

Figure 4: Linear contract's IC and IR constraints for the binary effort model.

This may seem odd since the firm could transfer the entire firm to the manager. However, the CRRA model involves wealth effects where the monetary cost of effort increases with wealth (supposedly as a function of the manager's alternative occupations). With an equity payment, the increase in performance sensitivity to high outcomes, which is necessary given that the likelihood ratio involves information at the upper tail, transfers significant wealth with little incentive value for lower outcomes – since the manager can retain the equity. This, in turn, increases total wealth and makes it difficult to elicit effort. If the manager were given the entire firm, the monetary cost of effort would render it inefficient even if contractible. By contrast, option-based compensation reduces payments for low outcomes and thus provides incentives without excessively increasing wealth. In summary, the optimality of option-based compensation can be linked back to theory, present in Rogerson 1985a, that a manager with lesser wealth is more willing to work but is stated here in terms of contract form rather than an inter-temporal trade-off.

For CARA, the manager can be motivated to provide effort with a share contract because the cost of effort is constant "in utility" and does not increase if one raises payments. However, it also comes with a potentially significant immediate dilution and would require an ownership of approximately 31% of Tesla. Given his initial stake of 21.9%

4.7 Continuous effort model

In this section, we investigate estimating the contract's underlying parameters when the manager's effort is continuous. As explained in section 3, the continuous effort model is compatible with unbounded compensation, as is effectively observed. The interpretable estimate given by the model is the relative productivity of effort Δ_c representing the effect on output for an increase in effort obtained at a unit dollar cost. We focus here on comparing these estimates with the ones found using the binary effort model. The equivalent object from previous estimates is the ratio $\frac{\Delta}{C}$.

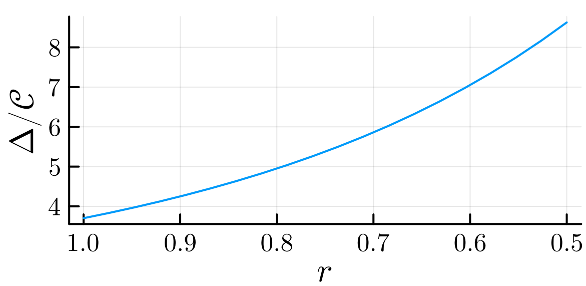
Table 6 provides both estimates for a near-risk-neutral manager. Interestingly, the per-dollar costs of output estimated using both models are virtually identical. While the use of a continuous effort model and the limitations on observables restrict our ability to recover other objects of interest, such as the full counterfactual distribution or the manager's reservation wage, these results suggest that both models are consistent in the recovered contract parameters.

Figure 5 confirms that this consistency not only applies to the risk-neutral limit but also when one assumes that the agent is risk-averse. As with previous interpretations, the increase in contribution per dollar cost of effort may initially seem surprising. As shown above, with

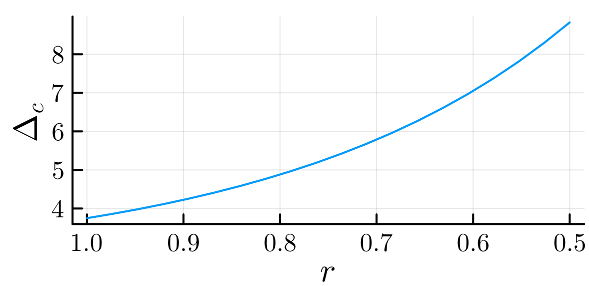
Model	Description	CRRA RN limit	CARA RN limit
Binary effort	$\Delta/\mathcal{C}$	\$ 3.75	\$ 3.51
Continuous effort	Δ_c	\$ 3.85	\$ 3.46

Note: Tabulates the contract estimates of the contribution of effort to output per dollar cost of effort for CRRA and CARA utilities, assuming that the manager is near-risk-neutral. The first row provides the ratio of the total contribution of effort to output (Δ) by the total cost of effort ($\mathcal{C}$) using the binary effort model presented in section 2. The second row provides the marginal contribution of effort to output per marginal cost of effort (Δ_c) derived from the continuous effort model presented in section 3.

Table 6: Contribution of effort to output for binary and continuous effort models.



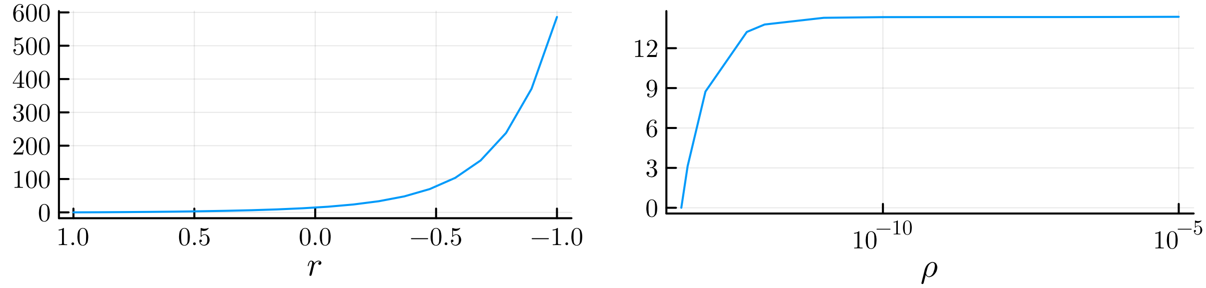
(a) Binary effort



(b) Continuous effort

Plots the effect of effort on output per dollar cost of effort as a function of risk-aversions assuming a CARA utility function. Plot (a) is based on the binary effort model presented in section 2, and Plot (b) is based on the continuous effort model presented in section 3. These estimates consider the full vesting conditions (market capitalization and operational milestones) using the calibrated parameters from table A.1. The manager tends toward risk-neutrality as $r \to 1$.

Figure 5: Productivity of effort per dollar cost as a function of risk-aversion.



(a) CRRA

(b) CARA

Plots the validity condition for the continuous effort model stated in corollary 1 as a function of risk-aversion for CRRA and CARA utilities. The validity condition is computed as $\mathbb{E}\left(\frac{1}{u'(w(\mathbf{x}))}\right) - \frac{1}{u'(x(\hat{x}))}$, where $\hat{x} = \$3.61B$ corresponds to the peak for the $f(\mathbf{x}|1)$ distribution.

Figure 6: Validity of risk-aversion for the continuous effort model.

the binary effort model, estimates for both the cost of effort and total contribution decrease in risk-aversion because the observed contract remains fixed. Figure 5 illustrates that the *recovered* share of the contract payment covering the cost of effort reduces faster than the decrease in output. In other words, taking the contract parameters as given, while risk-aversion increases the cost of inducing effort for the principal, the amount of output created by inducing such effort does not increase as much.

The theoretical analysis in section 3.3 allows us to investigate restrictions on plausible risk-aversion coefficients in the context of the continuous effort model. Following the result in corollary 1, the manager's preference must satisfy equation 30. Figure 6 plots the difference between both sides of equation 30. Interestingly, it shows that the only valid risk-aversion levels for CRRA and CARA utilities are when they tend to risk-neutrality. These results further solidify the value of deriving and recovering contract estimates at these limits, at least in the case of Elon Musk's 2018 contract.

5 Concluding Remarks

A fundamental unsolved problem in incentive theory lies in its lack of transparency. Although theoretical models often offer many predictions about the optimal design of incentives, designing executive contracts in practice often relies on an unstructured process where compensation consultants and boards reflect on a high-dimensional problem top executives face. While we recognize that the approach can be productive in holistically understanding incentives without binding the contract to a specific mathematical framework, it poses apparent issues when attempting to explain the fairness and desirability of potentially costly compensation to superstar CEOs. Under an informal analysis, for example, it would be almost impossible to explain the compensation's magnitude or whether the contract's underlying hypotheses are plausible. We chose a recent decision as a working example, making it clear that the problem is not just theoretical. In Tornetta v. Musk, the court ruled that the unstructured process that the board followed in selecting the compensation failed to demonstrate the business purpose of the incentives and, with no attempt at quantification, that it had been unable to (scientifically) defend why the contract had been chosen.

Our approach offers an alternative polar view by considering a restrictive effort problem whose solution is a mathematically well-posed problem. Within this framework, a contract and distribution of performance measures uniquely map to the parameters of the agency problem and offer insights as to whether these parameters may be reasonable to shareholders. As with any approach relying on revealed preferences, this does not mean that these parameters are true; for example, the board may exaggerate the cost of effort, its contribution to incentives, or set an excessive CEO surplus. However, evaluating these inferred parameters for plausibility remains much easier than considering verbal arguments about the necessity of incentives. Further, of particular relevance to the case of superstar CEOs, we offer new interpretable characterizations near risk-neutrality, which is likely reasonable for executives choosing to retain a considerable number of shares and with a significant amount of personal wealth. We show that this type of preference cannot be rejected for most realistic contracts. Hence, it is quite possible that the agency cost, i.e., the supplementary risk premium required to induce effort and which has been the object of most of the literature (Holmström 1979), is minimal for such executives once incentives are appropriately resolved.

Of course, a restrictive approach based on strong assumptions is not designed to be a serious attempt at capturing features of the problem in an abstract model but is a literal representation of the effort problem. The hidden action paradigm allows us to phrase the problem in terms of a simple economic mechanism design and, as we argue here, any generalization in terms of other informational asymmetries would likely encounter significant challenges to identification, and it is an open question as to whether more comprehensive mathematical models can be empirically assessed. To this extent, we hope that our model will contribute as steps toward a scientific approach to compensation.

Appendices

A Calibration

We detail here the calibration of the accounting processes defined in equation 31. We initialize the processes with a market capitalization of $p_0 = 350.02 \times 173,017,565$ shares $= \$60.56B$, which corresponds to the common practice of setting the strike equal to the current market price, and initialize rev_0 and eb_0 (excluding option compensation) to the 2017 revenue and EBITDA, respectively. This implies starting values for the ratios of Revenue-to-price and EBITDA-to-price of 0.224 and 0.006, respectively.[17] In Table A.1, the implicit volatility of Tesla's returns σ_p and the risk-free rate r_f are obtained from the values reported in Tesla's proxy statement to value the grant. The drift μ_p is derived from the CAPM with β being estimated using monthly returns and value-weighted market returns over the previous five years, from December 2013 to December 2017, included. We obtain the variances of the ratios of revenue-to-price and EBITDA-to-revenue and their covariances using historical quarterly data from Compustat between December 2013 and December 2017, included. The drifts of these ratios are estimated using analysts' forecasts from IBES. Due to the limited availability of market capitalization forecasts, it is not practical to directly estimate the drift of the revenue-to-price and EBITDA-to-Revenue ratios. Instead, we calibrate these drifts such that the simulated drifts of Revenues and EBITDA match the observed analyst's forecasts.

We obtain the EBIT drift[18] from the median forecast of analysts over a 5-years horizon 2018-2022, made between July 2017 and the start date of the contract, January 21, 2018.[19] Similarly, we obtain median revenue forecasts for 2018-2022 under the same window. We then construct revenue and EBITDA multiples from the assumed processes from equation 31, recovering the drift as the average change in revenue multiples that match the forecasted drifts of revenue and EBITDA. Table A.2 compares the median drifts forecasted by analysts and those generated by the model using the drifts of the multiples listed in table A.1. While the drift under the objective probability measure (μ_p) is necessary to calibrate the

[17] 2017 Revenue is defined as the sum of Tesla's 2017 revenues from their quarterly financial statements. Tesla does not directly report EBITDA, so we compute quarterly EBITDA as sales minus cost of goods sold minus SG&A expenses. 2017 EBITDA is then the sum of 2017 quarterly EBITDAs.

[18] There are no available long term analysts forecasts of total EBITDA and only one forecast of EBITDA per share.

[19] Specifically, we compute the median annual EBIT and Revenue change across all analysts. We do not adjust the EBIT forecasts for depreciation and amortization or net out the stock option expense because these are not forecasted. However, as long as the depreciation and amortization and the stock option expense increase in proportion to the EBIT, this will not affect our analysis.

accounting processes, all analyses are based on simulations using the drift under the risk-neutral probability measure (r_f).

Assumptions			
Risk-free interest rate	r_f	2.64%	Proxy statement
Risk-premium		4.77%	Damodaran estimate of historical risk-premium in 2017
CAPM β	β	1.29	Estimates with monthly return and value-weighted market return over the previous 5 years
Expected stock return	μ_p	8.77%	CAPM $r_f + \beta(E[r_m] - r_f)$
Stock volatility	σ_p	45.35%	Proxy statement
Revenue-Price ratio drift	μ_r	0.164	IBES analysts forecasts
Revenue-Price ratio volatility	σ_r	4.72%	Compustat
EBITDA-Revenue ratio drift	μ_e	0.011	IBES analysts forecasts
EBITDA-Revenue ratio volatility	σ_e	6.4%	Compustat
Covariance Price and Revenu-Price	$cov(W_t^p, W_t^r)$	-1.813	Compustat
Covariance Price and EBITDA-Revenue	$cov(W_t^p, W_t^e)$	0.184	Compustat
Covariance Revenu-Price and EBITDA-Revenue	$cov(W_t^r, W_t^e)$	0.476	Compustat
Starting Values			
Market capitalization	p_0	\$ 60.56 B	173,017,565 shares at price 350.02. As of contract date.
Revenue-Price ratio	rev_0/p_0	0.224	Compustat. As of December 31, 2017
EBITDA-Revenue ratio	eb_0/rev_0	0.006	Compustat. As of December 31, 2017

Note: Tabulates the parameters used to simulate the processes for prices p_t, revenue-to-price ratio $\frac{rev_t}{p_t}$ and EBITDA-to-revenue ration $\frac{eb_t}{rev_t}$ described in equation 31.

Table A.1: Calibration values

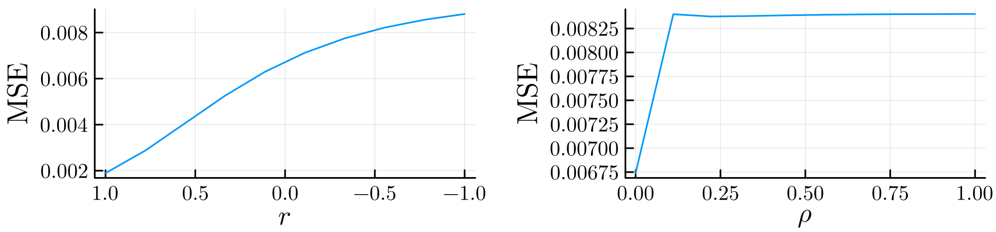
Drift	IBES	Model	Std. Error
μ_{rev}	7818.9	7950.09	21.11
μ_{eb}	1090.6	1110.61	5.28

Note: Compares the median drifts forecasted by analysts and those generated by the model using the drifts of the multiples listed in table A.1. The model estimates and standard errors are computed by simulating 10,000 firms over ten years and bootstrapped 100 times.

Table A.2: Drifts estimation

B Complementary Analysis

B.1 Parametric estimation



(a) CRRA (b) CARA

Plots the minimum mean-square error (MSE) for the parametric estimation of $f(\mathbf{x}|0)$ for different levels of risk-aversions. For a given risk-aversion (r or ρ), the parameters for $f(\mathbf{x}|0)$, (μ_0, σ_0), that minimize the MSE are found. The value of the MSE as a function of risk-aversion is plotted.

Figure B.1: Mean-square error for different levels of risk-aversion.

C Additional Discussions

C.1 Discrete Efforts.

Suppose that the manager chooses over multiple actions $e \in E$ and the firm solves, for each effort, the cost-minimizing contract (Grossman and Hart 1992). Let us label the elicited effort to $e^* = 1$ and the corresponding cost of effort $A(1) \equiv A$. The only difference in this setting is that the incentive constraint must be replaced by the best possible deviation:

$$\int Au(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} \geq \max_e \int A(e)u(w(\mathbf{x}))f(\mathbf{x}|e)d\mathbf{x}. \tag{35}$$

Generically, there should be a unique optimal effort e' that maximizes the right-hand side, which implies that the analysis carries over to this setting except that the cost of effort $A/A(e')$ and the manager surplus $\underline{w}/A(e')$ are now obtained relative to binding effort deviation.

C.2 Noisy high performance.

The identification of the parameters of the model relies on the existence of an observable performance level $\overline{x}$ consistent with high effort. However, this event may be improbable enough to be observed empirically, or if $f(\mathbf{x}|0)/f(\mathbf{x}|1)$ is bounded away from zero, it may not exist. In what follows, suppose that for sufficiently high performance, $f(\overline{\mathbf{x}}|0)/f(\overline{\mathbf{x}}|1) > 0$. Given that $f(\overline{\mathbf{x}}|0)/f(\overline{\mathbf{x}}|1)$ is not observable, this implies that the cost of effort to the manager can be bounded from below, as stated next.

Corollary 2 *For any $f(\overline{\mathbf{x}}|0)/f(\overline{\mathbf{x}}|1) > 0$, denoting $\overline{\gamma} \equiv 1/u'(w(\overline{\mathbf{x}})$, then $\overline{A} \leq A < 1$ (resp., $\overline{A} > A \geq 1$) if $u(.) > 0$ (resp., $u(.) < 0$), where:*

$$\overline{A} \equiv \frac{1}{\overline{\gamma} - \alpha}(\overline{\gamma} - \frac{\int f(\mathbf{x}|1)\frac{u(w(\mathbf{x}))}{u'(w(\mathbf{x}))}d\mathbf{x}}{\beta}). \tag{36}$$

C.3 Additive Effort.

Consider next the problem with an additive cost of effort, setting $u(x;1) \equiv u(x) - A$. This formulation follows Holmström 1979 and would imply that a wealthy manager can be more difficult to motivate (Rogerson 1985a). However, note that it no longer nests exponential preferences with the cost of effort in monetary terms, and the cost of effort is now expressed in fixed utils. The next Corollary develops the main result for this type of preference.

Corollary 3 *Under additive efforts,*

$$A = \frac{\gamma\beta - \int \frac{u(w(\mathbf{x}))}{u'(w(\mathbf{x}))} f(\mathbf{x}|1)d\mathbf{x}}{\gamma - \alpha} - \beta. \tag{37}$$

Under HARA preferences with a finite, $A \to a(\frac{\int w(\mathbf{x}) \ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x})})f(\mathbf{x}|1)d\mathbf{x}}{\int \ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x})})f(\mathbf{x}|1)dx} - W)$ as the manager becomes risk-neutral, and under constant absolute risk-aversion, $A \to 0$.

Corollary 3 adapts the main result to additive effort and yields similar insights. The cost of effort is now obtained as a difference rather than ratio in (37) and, in the limit, converges to a similar expression, which must be divided by a to interpret it in monetary terms.

C.4 Non-binding participation.

One possibility is that the compensation plan has to prescribe positive payouts, as a common practice in executive compensation is to avoid committing a manager to put additional personal wealth into the contract. Under this condition, all incremental incentives beyond regular stock ownership must be given as upside pay. To model this restriction, we assume next that $w(\mathbf{x}) \geq \tau(\mathbf{x})$ where $\tau(\mathbf{x})$ reflects an existing ownership and assume that this is sufficient to induce participation, as is common in this type of model (Innes 1990; Baldenius, Glover, and Xue 2016).

Corollary 4 *If the contract is subject to positive payments and let $u(.) \geq 0$, $w(\mathbf{x}) \geq \tau(\mathbf{x})$, and (IR) does not bind:*

$$A \leq \frac{\gamma u(\overline{\tau})}{\int f(\mathbf{x}|1)\frac{u(w(\mathbf{x}))}{u'(w(\mathbf{x}))}d\mathbf{x} + u(\overline{\tau})\int f(\mathbf{x}|1)(\gamma - \frac{1}{u'(w(\mathbf{x}))})d\mathbf{x}}. \tag{38}$$

where $\overline{\tau} = \max \tau(\mathbf{x})$.

When $w(\mathbf{x}) = \tau(\mathbf{x})$, the contract no longer reveals information about the incentive problem. As a result, information is lost about the assumed counter-factual distribution of effort $f(\mathbf{x}|0)$ and, therefore, the severity of the incentive problem. We can nevertheless observe that because the firm would have wanted to pay *below* $\tau(\mathbf{x})$, the probability of such binding outcomes under $e = 0$ must be large enough, which implies the set identification in (38) such that the effort must be sufficiently costly.[20]

C.5 Risk-adjusted Cost.

In the model, the "firm" stands for value to well-diversified investors. Because the performance measure $\mathbf{x}$ will typically contain non-diversifiable risk such that the value of $\mathbf{x} - w(\mathbf{x})$ to an investor is less than its expected value. For example, if $w(\mathbf{x})$ contains equity, the compensation cost is equivalent to a portfolio of contingent claims and should be discounted at the appropriate expected return. To address this, we assume that the contractible measure has two components $\mathbf{x} = (\mathbf{s}, \mathbf{y})$, such that $\mathbf{s}$ is a non-diversifiable risk that does not depend on effort (e.g., market movements) drawn from $f_s(.)$ and $\mathbf{y}$ is a firm-specific risk draw from $f_y(.|e, \mathbf{s})$, in short-hand $f_y(.|e)$. The firm's objective then needs to be adjusted to the risk-neutral probability measure $h(\mathbf{s})$, which, written in terms of the pricing kernel $m(\mathbf{s}) = h(\mathbf{s})/f_s(\mathbf{x})$, implies a formulation:

$$\int m(\mathbf{s})f(\mathbf{x}|1)(\phi(\mathbf{x}) - w(\mathbf{x}))d\mathbf{x}.$$

Note that the manager's incentive and participation constraints (IC) and (IR) remain unchanged (i.e., are based on $f_s(.)$) because the expected utility function is already explicitly written in terms of ex-post payoffs.[21] The same analysis as in the baseline thus applies to this

[20] The result is symmetric for $u(.) < 0$ and such that the condition must then be a lower bound, with the same interpretation.

[21] See Bertomeu 2015 for an analysis of the optimal contract with general preferences with state prices. Fischer 2000 and Margiotta and Miller 2000 show that, under constant absolute risk-aversion, the solution

setting, but after adjusting all terms $1/u'(w(\mathbf{x}))$ by $m(\mathbf{s})/u'(w(\mathbf{x}))$, so the inverse marginal utility is risk-adjusted.

C.6 Ownership constraints.

We have so far assumed that the manager could, in principle, costlessly divest all ownership, whether previously owned equity or equity received in the contract. However, there are institutional reasons why such divesting may not always be possible or desirable, either because the market is not sufficiently liquid and such sales would depress prices, the manager has external psychological reasons to keep the shares (e.g., a founder deriving personal benefits, (Friedman and Heinle 2020), or certain clawback provisions, such as an accounting restatement, require continued ownership. While the diverse nature of these considerations makes them impractical to model, we use here a simplified approach such that $w(\mathbf{x}) \geq \tau(\mathbf{x})$, where $\tau(.)$ captures these constraints, is continuous and assumed to be known to the researcher. Suppose that there are events with binding limited liability, i.e., for $\mathbf{x} \in X$, $u(\tau(\mathbf{x})) = u(w(\mathbf{x})) = 0$, and the constraint does not bind at the level of compensation consistent with high effort $w(\overline{\mathbf{x}}) > \tau(\overline{\mathbf{x}})$. To remove one parameter, we shall further assume as a baseline that $\tau(\mathbf{x})$ is sufficient to satisfy participation, which is ensured by $\mathbb{E}(u(\tau(\mathbf{x}))) \geq u(\underline{w})$.

Corollary 5 *For any contract, the cost of effort must satisfy:*

$$A \leq \frac{1}{\int_{N_0^c} f(\mathbf{x}|1)u(w(\mathbf{x}))d\mathbf{x} - \int_{N_0^c} f(\mathbf{x}|1)\frac{u(w(\mathbf{x}))}{\gamma u'(w(\mathbf{x}))}d\mathbf{x} + \int_{N_0} f(\mathbf{x}|1)(1 - \frac{1}{\gamma u'(w(\mathbf{x}))})d\mathbf{x}} \tag{39}$$

where $N_0 \equiv \{\mathbf{x} : w(\mathbf{x}) = \tau(\mathbf{x})\}$, *with complement* N_0^c.

Intuitively, a lower bound limits our ability to infer information about $f(.|0)$ when $\tau(x) = w(\mathbf{x})$, and we can only infer that the firm could have a lower payment based on agency considerations alone. As a result, the effort cost must be at least worse than if these constrained payments had been optimal.

under these assumptions involves an additively separable effect of the non-diversifiable risk, implying that the performance measure can be written net of market risk (for the case of equity). Instead of using state prices, we could equivalently write the firm's utility in terms of the preference of a representative manager, say, $v(\mathbb{E}(\phi(\mathbf{x}) - w(\mathbf{x})|s))$, diversifying the $\mathbf{y}$ component; see Cochrane 2005.

D Omitted Proofs

Proof of Proposition 1: The constraints (IC) and (IR) must bind; otherwise, the firm would be overpaying the manager or imposing too much risk. The Lagrangian is

$$\mathcal{L} = \int (\phi(\mathbf{x}) - w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} + \lambda(\int Au(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} - u(\underline{w}))$$

$$+\mu(\int Au(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} - \int u(w(\mathbf{x}))f(\mathbf{x}|0)d\mathbf{x}),$$

where λ and μ are Lagrange multipliers. Differentiating in $w(x)$ and rearranging the resulting first-order condition,

$$\frac{1}{u'(w(\mathbf{x}))} = \lambda A + \mu(A - \frac{f(\mathbf{x}|0)}{f(\mathbf{x}|1)}). \tag{40}$$

Evaluating the above at $\overline{\mathbf{x}}$ yields the following equality

$$\underbrace{\frac{1}{u'(w(\overline{\mathbf{x}}))}}_{=\gamma} = A(\lambda + \mu). \tag{41}$$

Multiplying both sides of (40) by $f(\mathbf{x}|1)$ and integrating yields

$$\underbrace{\int \frac{1}{u'(w(\mathbf{x}))} f(\mathbf{x}|1)d\mathbf{x}}_{=\alpha} = A(\lambda + \mu) - \mu. \tag{42}$$

Equations (41) and (42) can be solved to recover the Lagrange multipliers $\lambda = (\frac{1}{A} - 1)\gamma + \alpha$ and $\mu = \gamma - \alpha$, which can be substituted in (40) to obtain

$$\frac{1}{u'(w(\mathbf{x}))} = \gamma - \frac{f(\mathbf{x}|0)}{f(\mathbf{x}|1)}(\gamma - \alpha) \tag{43}$$

so that the implied density of the performance measure without effort is

$$f(\mathbf{x}|0) = f(\mathbf{x}|1)\frac{\gamma - \frac{1}{u'(w(\mathbf{x}))}}{\gamma - \alpha}. \tag{44}$$

Using this expression in the incentive constraint (IC) and solving for A,

$$A = \frac{\int u(w(\mathbf{x}))f(\mathbf{x}|0)d\mathbf{x}}{\int u(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x}} = \frac{\int u(w(\mathbf{x}))f(\mathbf{x}|1)\frac{\gamma - \frac{1}{u'(w(\mathbf{x}))}}{\gamma-\alpha}d\mathbf{x}}{\beta} = \frac{1}{\gamma-\alpha}(\gamma - \frac{\int f(\mathbf{x}|1)\frac{u(w(\mathbf{x}))}{u'(w(\mathbf{x}))}d\mathbf{x}}{\beta}), \tag{45}$$

such that $\beta \equiv \int u(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x}$. The remaining parameter $\underline{w}$ follows readily from (IR).$\square$

Proof of Proposition 2: We take the limit over the counter-factual distribution of effort in (44), assuming all limits below are well-defined (which will be shown by calculating them explicitly) can be decomposed as

$$\lim_{r\to 1}\frac{f(\mathbf{x}|0)}{f(\mathbf{x}|1)} \;=\; \underbrace{\lim_{r\to 1}\frac{r-1}{\gamma-\alpha}}_{\equiv 1/K_1}\lim_{r\to 1}\frac{\Lambda(\mathbf{x})}{r-1}, \tag{46}$$

with $\Lambda(\mathbf{x}) \equiv \gamma - \frac{1}{u'(w(\mathbf{x}))}$. From l'Hôpital rule,

$$\begin{aligned}
\lim_{r\to 1}\frac{\Lambda(\mathbf{x})}{r-1} &= \lim_{r\to 1}\frac{\partial}{\partial r}\{\frac{1}{u'(w(\overline{\mathbf{x}}))} - \frac{1}{u'(w(x))}\} && (47)\\
&= \lim_{r\to 1}\frac{1}{a}\underbrace{(b+\frac{aw(\overline{\mathbf{x}})}{1-r})^{1-r}}_{\to 1}(\underbrace{\frac{aw(\overline{\mathbf{x}})}{b(1-r)+aw(\overline{\mathbf{x}})}}_{\to 1} - \ln(b+\frac{aw(\overline{\mathbf{x}})}{1-r})) && (48)\\
&\quad -\frac{1}{a}\underbrace{(b+\frac{aw(\mathbf{x})}{1-r})^{1-r}}_{\to 1}(\underbrace{\frac{aw(\mathbf{x})}{b(1-r)+aw(\mathbf{x})}}_{\to 1} - \ln(b+\frac{aw(\mathbf{x})}{1-r})) && (49)\\
&= \lim_{r\to 1}\frac{1}{a}\ln(\frac{b+\frac{aw(\mathbf{x})}{1-r}}{b+\frac{aw(\overline{\mathbf{x}})}{1-r}}) = \frac{\ln(w(\mathbf{x})/w(\overline{\mathbf{x}}))}{a}. && (50)\\
\lim_{r\to 1}\frac{\gamma-\alpha}{r-1} &= \mathbb{E}(\lim_{r\to 1}\frac{\Lambda(\mathbf{x})}{r-1}) = \frac{\mathbb{E}(\ln(w(\mathbf{x})/w(\overline{\mathbf{x}})))}{a}. && (51)
\end{aligned}$$

The characterization in (12) follows readily from reinjecting these expressions in (46). The monetary cost of effort is then given by

$$\lim_{r\to 1}\mathcal{C} = \lim_{r\to 1}\beta - \beta A = \frac{\int (W - w(\mathbf{x}))\ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x})})f(\mathbf{x}|1)d\mathbf{x}}{\int \ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x})})f(\mathbf{x}|1)dx}, \tag{52}$$

given that $A = \mathbb{E}(u(w(\mathbf{x}))f(\mathbf{x}|0))/\beta$ and $\beta \to W$.$\square$

Proof of Proposition 3: Evaluating (2) using an exponential utility $u(w) = e^{-aw}$ and noting that $u(w)/u'(w) = -1/a$,

$$A = \frac{1}{\beta}\frac{\beta\gamma + 1/a}{\gamma - \alpha} \to 1, \tag{53}$$

because

$$\begin{aligned}
\lim_{a\to 0} 1/\beta &= -1 && (54)\\
\lim_{a\to 0} \beta\gamma + 1/a &= \lim_{a\to 0} \frac{1}{a}(1 - \int e^{-a(w(\mathbf{x})-w(\overline{\mathbf{x}}))} f(\mathbf{x}|1)d\mathbf{x}) && (55)\\
&= \lim_{a\to 0} \int \frac{a(w(\mathbf{x}) - w(\overline{\mathbf{x}})) + o(a^2)}{a} f(\mathbf{x}|1)d\mathbf{x} = W - w(\overline{\mathbf{x}}) && (56)\\
\lim_{a\to 0} \gamma - \alpha &= \lim_{a\to 0} \int (\frac{1}{u'(w(\overline{\mathbf{x}}))} - \frac{1}{u'(w(\mathbf{x}))}) f(\mathbf{x}|1)d\mathbf{x} = w(\overline{\mathbf{x}}) - W. && (57)
\end{aligned}$$

The counter-factual distribution can similarly be obtained as

$$\begin{aligned}
\lim_{a\to 0} \frac{f(\mathbf{x}|0)}{f(\mathbf{x}|1)} &= \lim_{a\to 0} \frac{\gamma - \frac{1}{u'(w(\mathbf{x}))}}{\gamma - \alpha}\\
&= \frac{w(\overline{\mathbf{x}}) - w(\mathbf{x})}{w(\overline{\mathbf{x}}) - W}.
\end{aligned}$$

To obtain the monetary cost of effort, note that:

$$\lim_{a\to 0} \mathcal{C} = \lim_{a\to 0} \frac{\ln A}{a} = \frac{\lim_{a\to 0} \frac{\partial A}{\partial a}}{\lim_{a\to 0} A} = \frac{\lim_{a\to 0}\{\frac{\partial \mathcal{U}}{\partial a}\mathcal{V} - \mathcal{U}\frac{\partial \mathcal{V}}{\partial a}\}}{(\lim_{a\to 0}\mathcal{V})^2}, \tag{58}$$

$$\text{where } \mathcal{U} \equiv \gamma - \int f(x|1)\frac{u(w(x))}{u'(w(x))}dx/\beta = \gamma + \frac{1}{a\beta} \quad \text{and} \quad \mathcal{V} \equiv \gamma - \alpha. \tag{59}$$

It is readily verified that:

$$\begin{aligned}
\frac{\partial \gamma}{\partial a} &= \frac{e^{aw(\overline{x})}(aw(\overline{x}) - 1)}{a^2} && (60)\\
\frac{\partial \alpha}{\partial a} &= \int \frac{e^{aw(x)}(aw(x) - 1)}{a^2} f(x|1)dx && (61)\\
\frac{\partial \beta}{\partial a} &= \int f(x|1)w(x)e^{-aw(x)}dx. && (62)
\end{aligned}$$

Taking limits as $a \to 0$,

$$
\begin{aligned}
\lim_{a\to 0} \mathcal{V} &= w(\overline{\mathbf{x}}) - W \qquad \text{from (56)} \\
\lim_{a\to 0} \frac{\partial \mathcal{V}}{\partial a} &= \lim_{a\to 0} \frac{\partial \gamma}{\partial a} - \frac{\partial \alpha}{\partial a} \\
&= \lim_{a\to 0} \frac{\int (e^{aw(\overline{x})}(aw(\overline{x})-1) - e^{aw(x)}(aw(x)-1))f(x|1)dx}{a^2} \\
&= \frac{1}{2}\int f(x|1)(w(\overline{x})^2 - w(x)^2)dx \\
\lim_{a\to 0} \mathcal{U} &= \lim_{a\to 0} \frac{\gamma a + 1/\beta}{a} \\
&= \lim_{a\to 0} \frac{\partial\{\gamma a + 1/\beta\}}{\partial a} \\
&= \lim_{a\to 0} \{ \frac{e^{aw(\overline{x})}(aw(\overline{x})-1)}{a} + \gamma - \frac{\int f(x|1)w(x)e^{-aw(x)}dx}{\beta^2} \} \\
&= \lim_{a\to 0} \{ e^{aw(\overline{x})}w(\overline{x}) - \frac{\int f(x|1)w(x)e^{-aw(x)}dx}{\beta^2} \} \\
&= w(\overline{\mathbf{x}}) - W \\
\lim_{a\to 0} \frac{\partial \mathcal{U}}{\partial a} &= \lim_{a\to 0} \frac{\partial \gamma}{\partial a} - \frac{a\frac{\partial \beta}{\partial a} + \beta}{a^2\beta^2} \\
&= \frac{e^{aw(\overline{x})}(aw(\overline{x})-1)}{a^2} - \frac{a\int f(x|1)w(x)e^{-aw(x)}dx + \beta}{a^2\beta^2} \\
&= \frac{\beta^2 e^{aw(\overline{x})}(aw(\overline{x})-1) - a\int f(x|1)w(x)e^{-aw(x)}dx - \beta}{a^2\beta^2} \\
&= \frac{\int f(x|1)M(a;x)dx}{a^2}
\end{aligned}
$$

where:

$$
\begin{aligned}
M(a;x) &= e^{aw(\overline{x})}(aw(\overline{x})-1)-aw(x)e^{-aw(x)}/\beta^2-1/\beta. \\
M_a(a;x) &= \frac{e^{-aw(x)}\left(\beta\left(e^{aw(x)}\frac{\partial\beta}{\partial a}+w(x)(aw(x)-1)\right)\right)}{\beta^3} \\
&\quad +\frac{e^{-aw(x)}\left(2aw(x)\frac{\partial\beta}{\partial a}+aw(\overline{x})^2\beta^3 e^{a(w(\overline{x})+w(x))}\right)}{\beta^3}\to 0 \\
M_{aa}(a;x) &= \frac{e^{-aw(x)}\left(\beta^2\left(e^{aw(x)}\frac{\partial^2\beta}{\partial a^2}+w(x)^2(2-aw(x))\right)-6aw(x)(\frac{\partial\beta}{\partial a})^2\right)}{\beta^4} \\
&\quad +\frac{\left(2aw(x)\frac{\partial^2\beta}{\partial a^2}-2e^{aw(x)}(\frac{\partial\beta}{\partial a})^2-4w(x)(aw(x)-1)\frac{\partial\beta}{\partial a}\right)}{\beta^3} \\
&\quad +\frac{w(\overline{x})^2\beta^4(aw(\overline{x})+1)e^{a(w(\overline{x})+w(x))}}{\beta^3} \\
&\to \lim_{a\to 0}\frac{\beta\frac{\partial^2\beta}{\partial a^2}+4w(x)\frac{\partial\beta}{\partial a}-2(\frac{\partial\beta}{\partial a})^2+\beta^3 w(\overline{x})^2+2\beta w(x)^2}{\beta^3} \\
&\to \frac{\int w(x)^2 f(x|1)dx+4w(x)W-2W^2-w(\overline{x})^2-2w(x)^2}{-1} \\
\lim_{a\to 0}\frac{\partial\mathcal{U}}{\partial a} &= \frac{1}{2}\lim_{a\to 0}\int f(x|1)M_{aa}(a;x)dx \\
&= \frac{1}{2}\int(w(\overline{x})^2+w(x)^2)f(x|1)dx-W^2 \\
\lim_{a\to 0}\frac{\partial A}{\partial a} &= \frac{\frac{1}{2}\int(w(\overline{x})^2+w(x)^2)f(x|1)dx-\frac{1}{2}\int f(x|1)(w(\overline{x})^2-w(x)^2)dx-W^2}{w(\overline{\mathbf{x}})-W} \\
&= \frac{\int(w(x)^2-W^2)f(x|1)dx}{w(\overline{\mathbf{x}})-W}.
\end{aligned}
$$

And, finally,

$$
\begin{aligned}
\underline{w} &= u^{-1}(\int u(w(\mathbf{x})f(\mathbf{x}|0)d\mathbf{x})=-\frac{1}{a}\ln(\int e^{-aw(x)}\frac{w(\overline{\mathbf{x}})-w(\mathbf{x})}{w(\overline{\mathbf{x}})-W}f(x|1)dx) \\
\lim_{a\to 0}\underline{w} &= \lim_{a\to 0}\frac{\int w(x)e^{-aw(x)}\frac{w(\overline{\mathbf{x}})-w(\mathbf{x})}{w(\overline{\mathbf{x}})-W}f(x|1)dx}{\int e^{-aw(x)}\frac{w(\overline{\mathbf{x}})-w(\mathbf{x})}{w(\overline{\mathbf{x}})-W}f(x|1)dx} \\
&= \int w(\mathbf{x})\frac{w(\overline{\mathbf{x}})-w(\mathbf{x})}{w(\overline{\mathbf{x}})-W}f(x|1)dx \\
&= W+\int w(\mathbf{x})\frac{W-w(\mathbf{x})}{w(\overline{\mathbf{x}})-W}f(x|1)dx \\
&= W-\frac{Var(w(\mathbf{x}))}{w(\overline{\mathbf{x}})-W}.\square
\end{aligned}
$$

Proof of Proposition 4: Under the risk-neutral limit in Proposition 2, the inequality can be written as

$$\begin{aligned}
0 &\leq \int (\phi(\mathbf{x}) - w(\mathbf{x}))f(\mathbf{x}|1)(1 - \frac{\ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x})})}{\int \ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x}')})f(\mathbf{x}'|1)d\mathbf{x}'})d\mathbf{x} \\
\Longleftrightarrow 0 &\leq \int (\phi(\mathbf{x}) - w(\mathbf{x}))f(\mathbf{x}|1)(\ln(w(\mathbf{x})) - \int \ln(w(\mathbf{x}'))f(\mathbf{x}'|1)d\mathbf{x}')d\mathbf{x} \\
\Longleftrightarrow 0 &\leq \int (\phi(\mathbf{x}) - w(\mathbf{x}))\ln(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} \\
&\quad - \int (\phi(\mathbf{x}) - w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} \int \ln(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x}')d\mathbf{x} \\
\Longleftrightarrow 0 &\leq cov(\phi(\mathbf{x}) - w(\mathbf{x}), \ln(w(\mathbf{x}))).
\end{aligned}$$

Similarly, under constant absolute risk-aversion, the inequality simplifies to

$$\int (\phi(\mathbf{x}) - w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} \geq \int \phi(\mathbf{x})f(\mathbf{x}|0)d\mathbf{x} - W + \lim_{a \to 0} \mathcal{C}, \tag{63}$$

which implies the inequality $0 \leq cov(\phi(\mathbf{x}) - w(\mathbf{x}), w(\mathbf{x})) - \frac{Var(w(x))}{w(\overline{\mathbf{x}}) - W}.\square$

Proof of Proposition 5: Let $\chi = \text{Sign}(u(w))$. The Lagrangian of the continuous effort problem is

$$\mathcal{L} = \int (\phi(\mathbf{x}) - w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} + \lambda(\int u(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} - u(\underline{w}))$$

$$+\mu \int u(w(\mathbf{x}))f(\mathbf{x}|1)(\frac{f_e(\mathbf{x}|1)}{f(\mathbf{x}|1)} - \chi)d\mathbf{x},$$

where λ and μ are the Lagrange multipliers associated to (IR2) and (IC2). To prove the statement, we need to find valid coefficients $(\lambda, \mu, \underline{w})$ that satisfy the local optimality conditions and the constraints such that the Lagrange multiplier λ is non-negative. Differentiating in $w(x)$ and rearranging the resulting first-order condition,

$$\frac{1}{u'(w(\mathbf{x}))} = \lambda + \mu(\frac{f_e(\mathbf{x}|1)}{f(\mathbf{x}|1)} - \chi). \tag{64}$$

Taking expectations yields the following analog to (42):

$$\alpha = \int \frac{1}{u'(w(\mathbf{x}))}f(\mathbf{x}|1)d\mathbf{x} \quad = \quad \lambda - \chi\mu, \tag{65}$$

which can be substituted in (64) to obtain

$$\frac{1}{u'(w(\mathbf{x}))} = \alpha + \mu \frac{f_e(\mathbf{x}|1)}{f(\mathbf{x}|1)}. \tag{66}$$

Solving for $f_e(\mathbf{x}|1)$ and reinjecting into the incentive constraint (IC2):

$$0 = \int u(w(\mathbf{x}))(\frac{1}{\mu}(\frac{1}{u'(w(\mathbf{x}))} - \alpha) - \chi) f(\mathbf{x}|1) d\mathbf{x} = -\beta(\frac{\alpha}{\mu} + \chi) + \frac{1}{\mu}\int \frac{u(w(\mathbf{x}))}{u'(w(\mathbf{x}))} f(\mathbf{x}|1) d\mathbf{x}, \tag{67}$$

which can be re-arranged to obtain μ:

$$\mu = \int \frac{\chi}{u'(w(\mathbf{x}))}(\frac{u(w(\mathbf{x}))}{\beta} - 1) f(\mathbf{x}|1) d\mathbf{x} = \frac{1}{\beta} cov(\frac{1}{u'(w(\mathbf{x}))}, u(w(\mathbf{x}))) > 0. \tag{68}$$

To show that the Lagrange multiplier λ is positive, let us substitute μ from (68) in (65),

$$\lambda = \alpha - \mu = \int \frac{u(w(\mathbf{x}))}{u'(w(\mathbf{x}))\beta} f(\mathbf{x}|1) d\mathbf{x} > 0. \tag{69}$$

It is then readily verified that $\underline{w} = u^{-1}(\beta)$, μ from (68) and $\lambda = \alpha + \mu$ satisfy (IR2) and (IC2).□

Proof of Proposition 6: (i) For the case of a finite and $r \to 1$,

$$\lim_{r\to 1} \frac{f_e(\mathbf{x}'|1)}{f(\mathbf{x}'|1)} = \frac{\lim_{r\to 1} \frac{1}{r-1}(\frac{1}{u'(w(\mathbf{x}'))} - \mathbb{E}(\frac{1}{u'(w(\mathbf{x}))}))}{\lim_{r\to 1} \frac{1}{r-1}(\mathbb{E}(\frac{u(w(\mathbf{x}))}{\beta u'(w(\mathbf{x}))}) - \mathbb{E}(\frac{1}{u'(w(\mathbf{x}))}))}, \tag{70}$$

The limit in the numerator is obtained using the same steps as (46)-(51):

$$\lim_{r\to 1} \frac{1}{r-1}(\frac{1}{u'(w(\mathbf{x}'))} - \mathbb{E}(\frac{1}{u'(w(\mathbf{x}))})) = \frac{\mathbb{E}(\ln(w(\mathbf{x}))) - \ln(w(\mathbf{x}'))}{a}. \tag{71}$$

For the term in the denominator, let us first rewrite

$$G \equiv \mathbb{E}(\frac{u(w(\mathbf{x}))}{\beta u'(w(\mathbf{x}))}) - \mathbb{E}(\frac{1}{u'(w(\mathbf{x}))}) \tag{72}$$

$$= \frac{1}{\beta}\mathbb{E}_{\mathbf{x},\mathbf{x}'}(\underbrace{\frac{b - br + aw(\mathbf{x})}{ar} - \frac{u(w(\mathbf{x}))}{u'(w(\mathbf{x}'))}}_{\equiv \Lambda(\mathbf{x},\mathbf{x}')}) \tag{73}$$

Using the same limit as in Proposition 2,

$$\lim_{r\to 1} \frac{\Lambda(\mathbf{x},\mathbf{x}')}{r-1} = \lim_{r\to 1} \frac{\partial \Lambda(\mathbf{x},\mathbf{x}')}{\partial r}$$

$$= \frac{(aw(\mathbf{x}) - br + b)^r (aw(\mathbf{x}') + b(-r) + b)^{-r} (b(r-1)(a(r+1)w(\mathbf{x}) + b) - aw(\mathbf{x}')(aw(\mathbf{x}) + b(r-1)r + b))}{(b(r-1) - aw(\mathbf{x}))ar^2}$$
$$-\frac{1}{ar}(aw(\mathbf{x}) - br + b)^r (aw(\mathbf{x}') - br + b)^{1-r} (\ln(aw(\mathbf{x}) - br + b) - \ln(aw(\mathbf{x}') - br + b)) - \frac{aw(\mathbf{x}) + b}{ar^2},$$

which can be evaluated as $r \to 1$ to obtain

$$\lim_{r\to 1} \frac{\Lambda(\mathbf{x}, \mathbf{x}')}{r-1} = w(\mathbf{x})(\ln(w(\mathbf{x}')) - \ln(w(\mathbf{x}))) \tag{74}$$
$$\lim_{r\to 1} G = W\mathbb{E}(\ln(w(\mathbf{x}))) - \mathbb{E}(w(\mathbf{x})\ln(w(\mathbf{x}))). \tag{75}$$

Regrouping (71) and (75) in (70) yields (27). Then,

$$\mathcal{M} = \frac{cov(\phi(\mathbf{x}), \ln(w(\mathbf{x})))}{cov(w(\mathbf{x})/W, \ln(w(\mathbf{x})))}. \tag{76}$$

Given that

$$\mathcal{C}'(1) = \frac{|\beta|}{u'(u^{-1}(\beta))} \to W, \tag{77}$$

it follows that $\Delta_c \to \mathcal{M}/W$ is given by equation (27).

(ii) Under constant absolute risk-aversion, let $w(\mathbf{x}) \neq W$,

$$\lim_{a\to 0} \frac{f_e(\mathbf{x}'|1)}{f(\mathbf{x}'|1)} = \frac{\lim_{a\to 0} \frac{1}{a}(\frac{1}{u'(w(\mathbf{x}'))} - \mathbb{E}(\frac{1}{u'(w(\mathbf{x}))}))}{\lim_{a\to 0} \frac{1}{a}(\mathbb{E}(\frac{u(w(\mathbf{x}))}{\beta u'(w(\mathbf{x}))}) - \mathbb{E}(\frac{1}{u'(w(\mathbf{x}))}))}, \tag{78}$$

where:

$$\lim_{a\to 0} \frac{1}{a}(\frac{1}{u'(w(\mathbf{x}'))} - \mathbb{E}(\frac{1}{u'(w(\mathbf{x}))})) = w(\mathbf{x}') - W$$
$$\lim_{a\to 0} \frac{1}{a}(\mathbb{E}(\frac{u(w(\mathbf{x}))}{\beta u'(w(\mathbf{x}))}) - \mathbb{E}(\frac{1}{u'(w(\mathbf{x}))})) = 0,$$

which establishes the divergence of $\frac{f_e(\mathbf{x}'|1)}{f(\mathbf{x}'|1)}$. Similarly,

$$
\begin{aligned}
\lim_{a\to 0}\Delta_c &= \frac{\lim_{a\to 0}\frac{1}{a}cov(\frac{-a\beta}{u'(w(\mathbf{x}))},\phi(\mathbf{x}))}{\lim_{a\to 0}\frac{1}{a}cov(\frac{1}{u'(w(\mathbf{x}))},|u(w(\mathbf{x}))|)} \\
&= \frac{-\beta\lim_{a\to 0}cov(\frac{1}{a}e^{aw(\mathbf{x})},\phi(\mathbf{x}))}{\lim_{a\to 0}\frac{1}{a}cov(\frac{1}{a}e^{aw(\mathbf{x})},e^{-aw(\mathbf{x})})} \\
&= \frac{\lim_{a\to 0}\frac{1}{a}cov(1+aw(\mathbf{x}),\phi(\mathbf{x}))}{\lim_{a\to 0}\frac{1}{a^2}cov(1+aw(\mathbf{x}),1-aw(\mathbf{x}))} \\
&= \frac{cov(w(\mathbf{x}),\phi(\mathbf{x}))}{-Var(w(\mathbf{x}))}\square
\end{aligned}
$$

Proof of Corollary 1: Evaluating (64) at $\hat{x}$ readily yields (30). To show that $\hat{\mathbf{x}}$ is the mode of the distribution, note that $f_e(\hat{\mathbf{x}}|1) = h_e(\hat{\mathbf{x}},1)g'(h(\hat{\mathbf{x}},1)) = 0$ implies $g'(h(\hat{\mathbf{x}},1)) = 0$. Then, $f_x(\hat{\mathbf{x}}|1) = h_x(\hat{\mathbf{x}},1)g'(h(\hat{\mathbf{x}},1)) = 0$, implying that $\hat{x}$ must be a peak of the distribution $f(.|1)$.$\square$

Proof of Corollary 3:

The Lagrangian with additive cost of effort is

$$
\mathcal{L} = \int(\phi(\mathbf{x}) - w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} + \lambda(\int u(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} - A - u(\underline{w}))
$$

$$
+\mu(\int u(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} - A - \int u(w(\mathbf{x}))f(\mathbf{x}|0)d\mathbf{x}),
$$

where λ and μ are Lagrange multipliers. Differentiating in $w(x)$ and rearranging the resulting first-order condition,

$$
\frac{1}{u'(w(\mathbf{x}))} = \lambda - \mu\frac{f(\mathbf{x}|0)}{f(\mathbf{x}|1)}. \tag{79}
$$

Evaluating the above at $\overline{\mathbf{x}}$ yields an equality $\gamma = \frac{1}{u'(w(\overline{\mathbf{x}}))} = \lambda$. Taking expectations on (79) yields $\mu = \gamma - \alpha$ and can be substituted in (79) to obtain

$$
\frac{f(\mathbf{x}|0)}{f(\mathbf{x}|1)} = \frac{\gamma - \frac{1}{u'(w(\mathbf{x}))}}{\gamma - \alpha}, \tag{80}
$$

which is identical to (2). Using this expression in the incentive constraint and solving for A,

$$
\begin{aligned}
A &= \int u(w(\mathbf{x}))f(\mathbf{x}|1)d\mathbf{x} - \int u(w(\mathbf{x}))f(\mathbf{x}|0)d\mathbf{x} && (81) \\
&= \frac{\gamma\beta - \int\frac{u(w(\mathbf{x}))}{u'(w(\mathbf{x}))}f(\mathbf{x}|1)d\mathbf{x}}{\gamma - \alpha} - \beta. && (82)
\end{aligned}
$$

Using similar steps as in Corollary 2, under the HARA class, as $r \to 1$,

$$A = \frac{\gamma\beta - \int \frac{u(w(\mathbf{x}))}{u'(w(\mathbf{x}))} f(\mathbf{x}|1)d\mathbf{x}}{\gamma - \alpha} - \beta \to a(\frac{\int w(\mathbf{x}) \ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x})}) f(\mathbf{x}\ |1)d\mathbf{x}}{\int \ln(\frac{w(\overline{\mathbf{x}})}{w(\mathbf{x})}) f(\mathbf{x}|1)dx} - W).$$

Under constant absolute risk-aversion, $A = \frac{\beta\gamma + 1/a}{\gamma - \alpha} - \beta \to 0$.

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