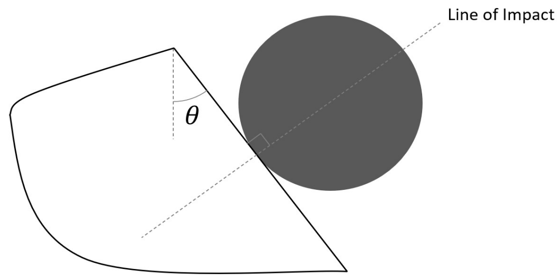
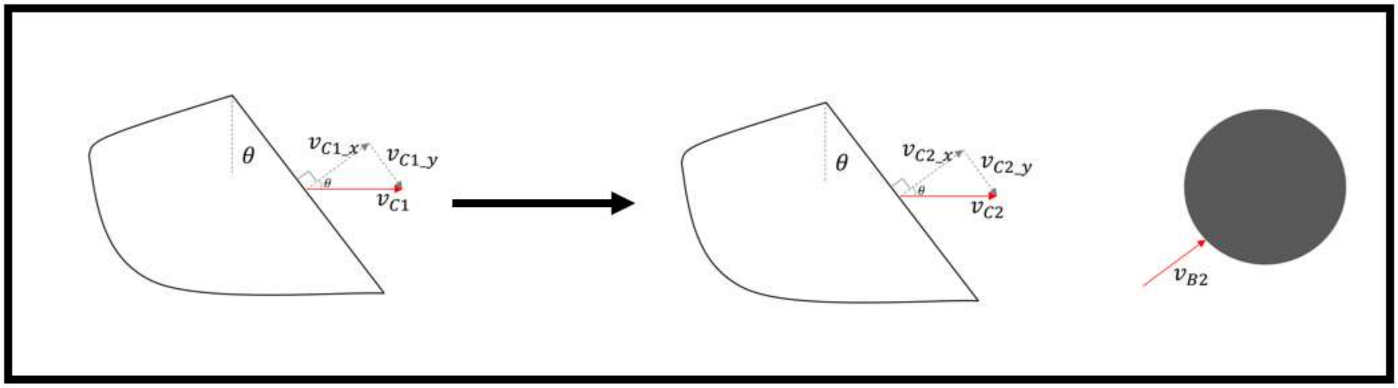
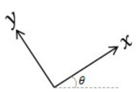


In order to analyse this system, we will need to use two principles that will lead us to 2 equations, enabling this problem to be solved - **Coefficient of Restitution** & **Conservation of Momentum**.

NOTE: In our analysis, we treat the golf club as an object separate from the swinging arm. This allows us to use Conservation of *Linear* Momentum instead of Conservation of *Angular* Momentum.





1) Conservation of Momentum (in x-direction):

$$m_C v_{C1\_x} + m_B v_{B1} = m_c v_{C2\_x} + m_B v_{B2}$$

2) Coefficient of Restitution (Along line of impact):

$$e := \frac{v_{B2} - v_{C2\_x}}{v_{C1\_x} - v_{B1}}$$

Example

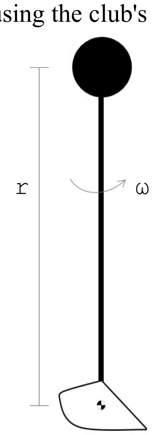
In this example, we will use the following input parameters:

$$\text{Club\_RPM} := 100\ \frac{\text{rotation}}{\text{min}} \qquad m_C := 1.49351\ \text{kg}$$

$$r := 1.088\ \text{m} \qquad m_B := 0.04590\ \text{kg}$$

$$e := 0.8 \qquad \theta := 30\ \text{deg}$$

First, using the club's angular velocity, we're able to find the linear velocity of the club head.



$$\omega := \text{Club\_RPM}\ \frac{1\ \text{min}}{60\ \text{s}}\ \frac{2 \cdot \pi\ \text{rad}}{1\ \text{rotation}} = 10.472\ \frac{\text{rad}}{\text{s}}$$

$$v_{C1} := r \cdot \omega = 11.3935\ \frac{\text{m}}{\text{s}}$$

In order to use Conservation of Momentum in the x-direction, the golf club's initial velocity has to be resolved in the x-direction.

$$v_{C1\_x} := v_{C1} \cdot \cos(\theta) = 9.8671 \ \frac{m}{s}$$

Before impact, we know that the golf ball is stationary.

$$v_{B1} := 0 \ \frac{m}{s}$$

As the masses of the club head and the ball is given, the Conservation of Momentum equation on the left (before impact) is now filled, giving us our first simultaneous equation.

$$m_C v_{C1\_x} + m_B v_{B1} = m_c v_{C2\_x} + m_B v_{B2}$$

$$(1.49351\ kg) \cdot (9.8671\ m/s) + (0.04590\ kg) \cdot (0\ m/s) = (1.49351 kg) \cdot (v_{C2\_x}) + (0.04590\ kg) \cdot (v_{B2})$$

$$\boxed{14.74 \ = 1.49351 \cdot v_{C2\_x} + 0.04590 \cdot v_{B2} \qquad < 1 >}$$

Using our equation for the Coefficient of Restitution, we obtain the second equation that can be used to solve the two final velocities:

$$e := \frac{v_{B2} - v_{C2\_x}}{v_{C1\_x} - v_{B1}}$$

$$\boxed{0.8 = \frac{v_{B2} - v_{C2\_x}}{9.8671} \qquad < 2 >}$$

Solving the equations <1> and <2>, we obtain theoretical values for the final velocities of the club and golf ball:

$$v_{B2} := 17.23 \ \frac{m}{s}$$

$$v_{C2\_x} := 9.34 \ \frac{m}{s}$$

Interpreting these results as losses and gains in momentum, we see the calculations below:

Golf Club Head: $$\boxed{Loss = m_c(v_{C2} - v_{C1}) = 1.49351\left(\frac{v_{C2\_x}}{\cos 30} - 11.3935\right) = -0.909\ kgm/s}$$

Golf Ball: $$\boxed{Gain = m_B(v_{B2} - v_{B1}) = 0.0459(v_{B2} - 0) = 0.791\ kgm/s}$$

As seen above, the momentum lost seems to exceed the momentum gained. This can be attributed to the loss in energy represented by the coefficient of restitution of 0.8. In the simulation however, it is expected that this will be further complicated by considering friction, angular momentum, etc.